Hanes' comp questions

3) Consider an economy where

$$y_t = E_t y_{t+1} - s r_t + \epsilon_t^{IS}$$
$$\pi_t = E_t \pi_{t+1} + y_t + \epsilon_t^{AS}$$
$$r_t = a\pi_t + b y_t + \epsilon_t^{mp}$$

where y is the output gap (the log of output minus the log of natural-rate output), r is the interest rate gap (the real interest rate minus the natural rate of interest), and π is the inflation rate. The ϵ 's are all uncorrelated with each other and mean-zero i.i.d. (*no persistence*). Expectations are rational.
Recall that we call a variable "procyclical" if it is positively correlated with the output gap; "countercyclical" if it is negatively correlated with the output gap, and "acyclical" if it is uncorrelated with the output gap.
For each case below, state whether each of the following variables is procyclical, countercyclical or acyclical: π , r, and the *nominal* interest rate i.

See the notes on NKIS/LM Simplest case: interest-rate rule, no persistence. (I said the shocks were i.i.d. and expectations are rational).Note that $E_t \pi_{t+1} = 0$ *always.*

The nominal interest rate is a bit tricky. Our usual definition for the real interest rate is $r_t = i_t - E_t \pi_{t+1}$. *On that definition the nominal interest rate is equal to the real interest rate. The answers I give below come from that assumption. But I didn't define the real interest rate here. It would not be unreasonable to define it as* $r_t = i_t - \pi_t$. *If you made that assumption, your answer to b) would be different with respect to the nominal interest rate (the cyclicality of the nominal interest rate would be ambiguous).*

3 pts each.

a) The variance of ϵ^{IS} is big, the variances of ϵ^{AS} and ϵ^{mp} very small. *A positive IS shock raises output, inflation and r. So all the variables are procyclical. (Nominal interest rate procyclical.)*
b) The variance of ϵ^{mp} is big, the variances of ϵ^{IS} and ϵ^{AS} very small. *A positive mp shock lowers output and inflation while raising r. So inflation is procyclical, r and i are countercyclical. (Nominal interest rate ambiguous.)*

c) The variance of ϵ^{AS} is big, the variances of ϵ^{mp} and ϵ^{IS} very small. *A positive AS shock lowers output, raises inflation and r. So all variables countercyclical.*

d) Does this model correspond to IS/LM, IS/MP, or neither? Explain. *IS/MP, because there's an interest-rate rule, not a fixed money supply. All of the above statements can be illustrated on IS/MP graphs.*

4) Consider a model with a competitive labor market with a market-clearing nominal wage W per unit of labor. A representative-agent household maximizes

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\theta} C_t^{1-\theta} + \frac{1}{1-\nu} (M_t / P_t)^{1-\nu} - \frac{1}{1+\lambda} L_t^{1+\lambda} \right]$$

The agent's nominal wealth evolves as $A_{t+1} = M_t + (A_t + W_t L_t - P_t C_t - M_t)(1+i_t)$
At time t, the agent takes as given A_t , the nominal interest rate on bonds i_t , the wage W_t and the price level P_t , and chooses consumption, labor and his real money balance. Assume "certainty equivalence" holds, so that in the agent's optimization problem you take expected values of future variables to be equivalent to actual known values of future variables.

a) Starting from the Bellman equation, derive an equation that gives the agent's demand for real money balance $(M/P)_t$ as a function of consumption C_t and the nominal interest rate i_t. *3 pts.*

$$V_t = \text{Max}\left[\frac{1}{1-\theta} C_t^{1-\theta} + \frac{1}{1-\nu}(M/P)_t^{1-\nu} - \frac{1}{1+\lambda} L_t^{1+\lambda} + \beta E_t V_{t+1}\right]$$

$$\frac{\partial V}{\partial (M/P)_t} = (M/P)^{-\nu} + \beta E_t \frac{\partial V}{\partial A_{t+1}}\left(P - P(1+i_t)\right) = \ldots\ldots P(-i_t) = 0$$

$$\frac{\partial V}{\partial C_t} = C_t^{-\theta} + \beta E_t \frac{\partial V}{\partial A_{t+1}}\left(-(1+i_t)P_t\right) = 0$$

$$\text{so } \beta E_t \frac{\partial V}{\partial A_{t+1}} P_t = C_t^{-\theta}\frac{1}{1+i_t} = (M/P)^{-\nu}\frac{1}{i}$$

$$\Rightarrow (M/P)_t = C_t^{\theta/\nu}\left(\frac{i}{1+i}\right)_t^{-1/\nu}$$

b) Starting from the Bellman equation, derive C_t as a function of $E_t C_{t+1}$ and the "real interest rate" $r_t = i_t - E_t \pi_{t+1}$, using $P_{t+1} = (1+\pi_{t+1})P_t$ and an approximation. *3 pts.*

$$V_t = \ldots\ldots$$

$$\frac{\partial V}{\partial C_t} = \ldots\ldots = 0$$

$$\frac{\partial V}{\partial A_{t+1}} = \frac{\partial u}{\partial C_{t+1}} \cdot \frac{\partial C_{t+1}}{\partial A_{t+1}} \quad \frac{1}{P_{t+1}} \text{ holding fixed } A_{t+2}$$

Using certainty-equivalence,

$$0 = C_t^{-\theta} + \beta E_t C_{t+1}^{-\theta}\frac{1}{E_t P_{t+1}}\left(-(1+i_t)\right)P_t$$

$$\Rightarrow C_t = E_t C_{t+1}\left[\beta(1+i_t)\frac{1}{1+E_t\pi_{t+1}}\right]^{-1/\theta}$$

Approximation: $\frac{1+i_t}{1+E_t\pi_{t+1}} \approx 1 + (i_t - E_t\pi_{t+1}) = 1 + r_t$

$$\Rightarrow C_t = E_t C_{t+1}\left[\beta(1+r_t)\right]^{-1/\theta}$$

c) Suppose that long-run steady state consumption $\bar{C}$ grows at rate g (that is $\bar{C}_{t+1} = (1+g)\bar{C}_t$). Starting from your answer to b), derive the long-run steady state real interest rate $\bar{r}$. *3 pts.*

$$\bar{C}_t = \bar{C}_{t+1}\left[\beta(1+\bar{r})\right]^{-1/\theta}$$

$$\bar{C}_t = (1+g)\bar{C}_t\left[\beta(1+\bar{r})\right]^{-1/\theta}$$

$$\Rightarrow 1+\bar{r} = (1+g)^{\theta}\beta^{-1}$$ → or you can take logs of both sides, using $\ln(1+\bar{r}) \approx \bar{r}$, and get

$$\bar{r} = (1+g)^{\theta}\beta^{-1} - 1$$

[illegible]

$$\bar{r} \approx \theta g - \ln\beta$$

d) Suppose this economy is closed and there is no capital - output is produced from labor alone. Suppose also there is a government that purchases output. Let Y_t stand for output in a period, G_t stand for government purchases of output in the period, and $\gamma_t = (G/Y)_t$ stand for the share of government purchases in output. Assume that γ evolves as:

$\gamma_t = \rho\gamma_{t-1} + \epsilon_t$ *where* ϵ is mean-zero i.i.d. Starting with your answer to b), derive an equation that gives the log of output y_t as a function of $E_t y_{t+1}$, r_t , γ_t and *no other variables*. ($E_t\gamma_{t+1}$ should *not* be in there.) *3 pts.*

$Y_t = C_t + G_t$ so $C_t = Y_t - G_t$, $E_t C_{t+1} = E_t(Y-G)_{t+1}$

so answer to b) becomes

$$Y_t - G_t = E_t(Y-G)_{t+1}\left[\beta(1+r_t)\right]^{-1/\theta}$$

$$Y - G = Y\left(1-\frac{G}{Y}\right) = Y(1-\gamma)$$

so

$$(1-\gamma_t)Y_t = E_t\left[(1-\gamma_{t+1})Y_{t+1}\right]\left[\beta(1+r_t)\right]^{-1/\theta}$$

take logs, use approximation $\ln(1-\gamma) \approx -\gamma$

$$-\gamma_t + y_t = -E_t\gamma_{t+1} + E_t y_{t+1} - \frac{1}{\theta}\ln\beta - \frac{1}{\theta}r_t$$

$$\Rightarrow y_t = E_t y_{t+1} - \frac{1}{\theta}\ln\beta - \frac{1}{\theta}r_t + \gamma_t - E_t\gamma_{t+1}$$

and $E_t\gamma_{t+1} = \rho\gamma_t$ so...

$$Y_t = -\frac{1}{\theta}\ln\beta + E_t Y_{t+1} - \frac{1}{\theta} r_t + (1-\rho)\epsilon_t$$

e) Suppose there is a central bank in this economy. Prior to each period *t*, prior to the realization of ϵ for that period, the central bank can set a value for the period's money supply M_t or the period's real interest rate r_t. Which would be better, assuming the central bank has a conventional loss function (the type of loss function we usually assume for a central bank)? Explain, using a graph or graphs as appropriate. *4 pts. Recall the Poole paper. In this economy there are no money-demand shocks, but there are IS(spending) shocks. So it would be better to fix the money supply. See graphs in notes.*

5) In the Diamond-Dybvig model of liquidity/financial crises, there are two "types" of consumers. How are the two types different? *6 pts.*

There are "impatient" and "patient" consumers. They have different utility functions. Impatient consumers get utility from consumtin in the first period, but no utility from consumption in the second period. Patient consumers get utility from consumption in both periods.

It is incorrect to say that impatient consumers withdraw in the first period, patient consumers leave their deposits in the bamk until the second period. This is incorrect because, in the event of a liquidity crisis, patient consumers also try to withdraw in the first period.

It is also incorrect to say that impatient consumers "prefer" to consume in the first period. That phrase does not imply that impatient consumers get no utility from second-period consumptoin.

Some of you wrote a lot, telling me lots of irrelevant things about the model. Don't waste time like that.

4) Recall Romer's static imperfect-competition model. The utility function for the representative household consuming a continuum of goods *j* is:

$$U = C - \frac{1}{\gamma}L^{\gamma} \text{ where } C = \left[\int_{j=0}^{1} C_j^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}$$

Recall each good is produced by a monopoly firm that hires labor from a competitive labor market at a market-clearing nominal wage *W*, producing one unit of output from each unit of labor input. Recall that η ends up being the elasticity of demand for an individual good: $C_j = (P_j / P)^{-\eta} C$.

a) As a function of the model's parameters, what is the socially optimal value of *L*, that is the level of employment per household that would be chosen by a social planner who acts to maximize the representative household's utility *U?*

3 pts.Socially optimal L maximizes the utility function given that C=L. The first-order condition gives $L = 1$ *(see notes).*

b) As a function of the model's parameters, what is the actual value of *L* in the model's "flex-price equilibrium," in which each household/firm's price is set at the profit-maximizing level, absent any type of nominal rigidity?

3 pts.See notes. Maximizing profit, each firm sets $P_j = \mu W$ *where* $\mu = \frac{\eta}{\eta-1} = \frac{1}{1-\eta}$ *. With all firms identical so that*

$P = P_j$,

that means $W / P = 1/\mu$.

Maximizing utility taking W and P as given, the household sets $L = (W / P)^{\frac{1}{\lambda-1}}$

Putting it all together, $L = (1/\mu)^{\frac{1}{\lambda-1}} = (1-\frac{1}{\eta})^{\frac{1}{\lambda-1}} < 1$

Note that if I say "as a function of the model's parameters," a answer with an endogenous variable on the right-hand side can't possibly be right.

c) Would it be optimal for the central bank in this economy to be run by a social planner who acts to maximize the representative household's utility as you assumed in a)? Explain.

4 pts. This is about "dynamic inconsistency of optimal monetary policy." From a), the desired level of output(employment) would be one. But the "natural rate" of output (employment) is less than one. If the central bank policymakers have a preference function that aims to keep output above the natural rate of output, inflation will (in long-run equilibrium, at least) be greater than desired inflation.

d) Why do all New Keynesian models generally have market power in product markets, like this one? Why not have *all* markets in the model be perfectly competitive?

4 pts. In a competitive market, a firm has an enormous incentive to adjust its price to the "optimal" price (which would be the competitive market-clearing price). Thus, if all markets were competitive, it would not be plausible to assume that a firm can, at least at times, fail to adjust its price to the optimal price.

It's important to mention the firm's incentive to adjust its price, or the idea that the profit ca be a smooth function of the price outside perfect competition.

6) We have seen models in which investment projects are controlled by "entrepreneurs" who must borrow capital from "investors" (or financial intermediaries).

a) In these models, there is an important "information asymmetry": something an entrepreneur knows, which an investor (or financial intermediary) cannot know unless he pays a cost. What is it?

3 pts. The realized outcome of the project (revenue or profit).

b) In these models there is a "financial accelerator." What does this phrase mean, and how does it work in these models?

3 pts. The phrase "financial accelerator" means "endogenous developments in credit markets work to propagate and amplify shocks to the macroeconomy" (Bernanke, Gertler and Gilchrist). Here, that means the asymmetric-information financial side of the model amplifies effects of an exogenous shock to real interest rates on output etc.

It works like this: an increase in the real interest rate reduces the value of entrepreneurs' assets, reducing their wealth, reducing their abioity to borrow from the investors - that is, tightening credit rationing and/or raising the required expected return on a project. That reduces investment spending and hence output.

c) In these models real activity can fall as a result of "debt deflation." What does this phrase mean, and how does it work in these models?

3 pts. "Debt deflation" means "redistributions between creditors and debtors arising from unanticipated price changes can have importnant real effects" (BGG again). Here, it works like this. The volume of investment and output depends partly on the value of entrepreneurs' real wealth, as mentioned above. As a deflation raises the real value of debt, it reduces borrowers' real wealth, raises lenders'. Entrepreneurs are borrowers. Hence deflation reduces their real wealth.

7) Let e stand for the rate of foreign exchange between dollars and pounds sterling, expressed as the number of pounds you can buy with a dollar. The expected change in the exchange rate is $(E_t e_{t+1} - e_t)$. The interest rate in the U.S. is i. The interest rate in Britain is j. Investors arbitrage between American assets and British assets to equalize the return to holding American assets and the return in dollars to holding British assets, so that:

$$i_t = j_t - (E_t e_{t+1} - e_t)$$

The American interest rate is known to follow an AR(1) process: $i_t = \rho_{US} i_{t-1} + \epsilon_t$ where ϵ is mean-zero i.i.d.

The British interest rate is known to follow an AR(1) process: $j_t = \rho_{UK} j_{t-1} + \nu_t$ where ν is mean-zero i.i.d. and ϵ is uncorrelated with ν. There is a known long-run steady-state value of e, denoted $\bar{e}$.

Derive an equation that gives e_t as a function of i_t and j_t.

7 pts.

$$i_t = j_t - E_t e_{t+1} + e_t$$

$$e_t = E_t e_{t+1} + i_t - j_t$$

Working back from $t = \infty$, $\bar{e}$, $e_{\infty-1} = \bar{e} + i_{\infty-1} - j_{\infty-1}$ ...

$$\Rightarrow e_t = \bar{e} + \sum_{\tau=1}^{\infty} (i-j)_{t+\tau}$$

$$= \bar{e} + \sum_{\tau=1}^{\infty} i_{t+\tau} - \sum_{\tau=1}^{\infty} j_{t+\tau}$$

$$= \bar{e} + \frac{1}{1-\rho_{US}} i_t - \frac{1}{1-\rho_{UK}} j_t$$