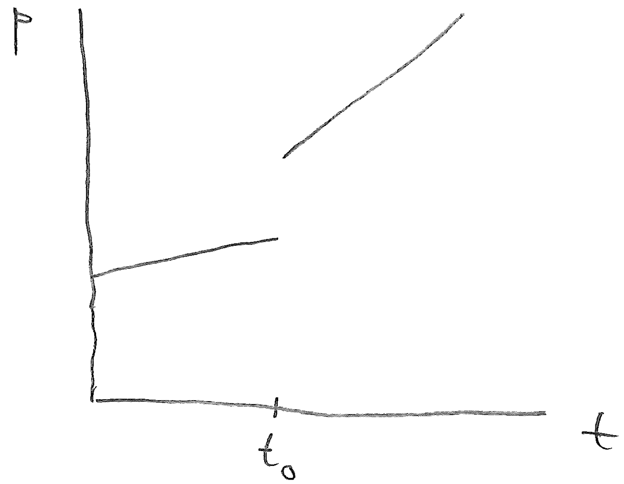
Hanes' comp questions June 2017

3) Suppose demand for log real money balances is given by $(m-p)^d = y - bi$ where y is log output and i is the nominal interest rate. The natural rate of output is fixed. The natural rate of interest is also fixed. Unexpectedly, at time t_0 , the rate of growth of the money supply increases from a lower value m_0 to a higher value m_1 . There is one path for the log price level p that will allow output to remain equal to the natural rate at all points in time. Draw this path on a graph. Put time on the horizontal axis of the graph. Mark time t_0 on the horizontal axis. *5 pts.*



4) a) Consider a new Keynesian expectations-augmented Philips curve of the form $\pi_t = E_t\pi_{t+1} + \beta y_t$, where y is the output gap. In one obvious way, this New Keynesian Phillips curve *plus* rational expectations is inconsistent with macroeconomic data. Explain, using equations. *5 pts.*

$$\pi_t = E_t \pi_{t+1} + \beta y_t \qquad \pi_{t+1} = E_t \pi_{t+1} + \varepsilon_{t+1}$$

$$\pi_{t+1} - \pi_t = -\beta y_t + \varepsilon_{t+1}$$

ϵ_{t+1} is the error in the time-t expected value for π_{t+1} . If expectations are rational, this error must be uncorellated with all information available at time t, which includes y_t . Thus, in data the change in inflation from t to t+1 must be equal to $-\beta y_t$ plus an uncorrelated disturbance ter,m. If you run a regression with $\Delta\pi_{t+1}$ on the left-hand side, and y_t on the right-hand side, the estimated coefficient should be zero. "High levels of output are associated with falls in inflation." Not true.

b) The Friedman-Phelps expectations-augmented Phillips curve is of the form $\pi_t = E_{t-1}\pi_t + \beta y_t$. In one obvious way, this Friedman-Phelps Phillips curve *plus* rational expectations is inconsistent with macroeconomic data. Explain, using equations. 5 pts.

$$\pi_t = E_{t-1} \pi_t + \beta y_t \qquad \pi_t = E_{t-1} \pi_t + \varepsilon_t$$

$$\varepsilon_t = \beta y_t \qquad y_t = \frac{1}{\beta} \varepsilon_t$$

ϵ_{t+1} is the error in the time-(t-1) expected value for π_t . If expectations are rational, this error must be uncorellated with all information available at time (t-1), which includes y_{t-1}. So the output gap in a period must be uncorrelated with the

output gap in the previous period. "No serial correlation in the output gap." Not true. Also, under rational expectations ϵ_{t+1} *is the error in the time-t expected value for* π_{t+1} *. If expectations are rational, this error must be uncorellated with all information available at time t, which includes* y_t $E_t[\epsilon_{t+1}]=0$ *, always. So* $E_t[y_{t+1}]=0$*, always. "No one ever forecasts that the economy will be in a boom, or a recession, in the upcoming period." Not true.*

5) Consider two economies, Wessex and Mercia. In both economies, $y_t = m_t - p_t$ where y is aggregate output, m is the log money supply and p is the log price level. In both economies, the money supply evolves as a random walk, that is $m_{t+1} = m_t + \epsilon_{t+1}$ where ϵ follows a standard normal distribution. The variance of ϵ is the same in both economies. In both economies, a firm must pre-specify the price that it will charge for the current period and the upcoming period, and pricesetting is "staggered": half the firms make their price plans in "even" periods, half make their plans in "odd" periods. The two economies are different in one way. In Wessex, a firm must charge the same price in both periods of its price plan. In Mercia, a firm can set different prices for the two periods of its plan. In which economy do recessions last longer? Explain. I am *not* asking for derivations here.

10 pts. Mercia is the Fischer model. Wessex is the Taylor model. In the Fischer model, a random-walk shock to aggregate demand affects output in the period of the shock and the following peeriod. In the Taylor model, such a shock affects output for many periods - the effect dies out in an AR(1) way. Thus, recessions last longer in Wessex. For full credit, you had to refer to these models, state that a shock affects output for only two periods in the Fischer model, and state that a demand shock affects output for many periods in Taylor.

6) Consider an economy in which business investment is subject to the "asymmetric information" problem described by the Romer textbook's model of "financial-market imperfections." “Entrepreneurs,” who control business projects, do not have enough wealth to fully fund projects; they need to borrow from “investors.” An investor is willing to provide funds to an entrepreneur as long as the expected return to the investment is greater than or equal to an exogenously determined return on safe bonds. An investor must pay a cost to observe the realized return on an entrepreneur’s business project. Suppose this economy is conquered by religious fanatics who ban the use of fixed-interest debt contracts (the type of debt contract described by the model). The only type of investment contract that can now take place is one in which an entrepreneur promises to pay an investor a *share* of the realized return to a project. How does this event affect the volume of business investment in this economy - increase, decrease, or no effect? Explain. I am *not* asking for derivations here.

10 pts. The expected value of the investor’s return to investing in a business project is the expected value of the payment from the entrepreneur minus the “expected verification cost” denoted A. A project is undertaken if:

Expected return to project $\geq$ *(1+r) + A*

where r is the return on safe bonds. The “expected verification cost” is the verification cost times the probability that the investor must pay the verification cost. Under the fixed-interest debt contract, the probability that the investor must pay the verification cost is less than one. Under the share contract, that probability is one. Thus A is larger under the share contract. Fewer projects will be undertaken, decreasing the volume of business investment, if share contracts replace debt contracts.

7) Consider a model like Romer's textbook static (one-period) model of imperfect competition. Each household i operates a monopoly firm and supplies labor to a perfectly competitive labor market. A household-firm does not use its own labor in production, but instead hires labor from the perfectly competitive labor market at a market-clearing nominal wage W per unit of labor. Each household acts to maximize:

$U_i = C_i - \frac{1}{\gamma} L_i^{\gamma}$ where C_i is an index of consumption of individual goods. Each household-firm's production function is $Y_i = H_i$, where H is the number of labor units the household-firm hires from the labor market. Demand for the good produced by a household-firm is $Y_i^D = (P_i / P)^{-\eta} Y$ where P is the price level, Y is average real income or real GDP *per household*, and $\eta > 1$.

Starting with these expressions *derive* what I called the "real rigidity equation," which gives the log of a firm's profit-maximizing price as a function of the log price level and the log of real GDP per household.

12 pts.

From profit maximization, derive an equation that gives the profit-maximizing price as a markup over the nominal wage W:

$$\pi_t = P_i Y_i^D - W Y_i = P_i (P_i / P)^{-\eta} Y - W (P_i / P)^{-\eta} Y$$

$$= P^{\eta} Y (P_i^{1-\eta} - P_i^{-\eta} W) \quad \text{Maximize term in parens}$$

$$0 = (1-\eta) P_i^{*-\eta} - (-\eta) P_i^{*-\eta-1} W$$

$$P_i^* = \frac{\eta}{\eta-1} W = \frac{1}{1-1/\eta} W$$

(You could also maximize real profit, get same answer.)

From utility maximization, derive an equation that gives a household's supplied quantity of labor as a function of the real wage:

$$U_i = C_i - \frac{1}{\gamma} L_i^{\gamma}$$

$$0 = \partial U_i / \partial L_i = \partial C_i / \partial L_i - L_i^{\gamma-1} = \frac{W}{P} - L_i^{\gamma-1}$$

$$\frac{W}{P} = L_i^{\gamma-1} \qquad W = L_i^{\gamma-1} P$$

which means the nominal wage is:

The number of firms is equal to the number of households, so $L_i = H_i$ *. Hence:*

With symmetry $L_i = H_i = Y_i = Y$

$W = Y^{\gamma-1} P$ Substitute into equation for P_i^*

$$P_i^* = \frac{\eta}{\eta-1} Y^{\gamma-1} P$$

$$\ln P_i^* = \ln(\eta/\eta-1) + \ln P + (\gamma-1) \ln Y$$

8) Consider two economies, Wessex and Mercia. In both economies $\pi_t = \beta E_t \pi_{t+1} + \lambda y_t$, where β is a time-discount factor between zero and one, and y is the output gap. Monetary policy satisfies the "Taylor principle." Expectations are rational. The output gap is known to evolve in an AR(1) process: $y_{t+1} = \rho y_t + \epsilon_{t+1}$ where ϵ follows a standard normal distribution. In Mercia, $\rho = 0.9$. In Wessex, $\rho = 0.6$. Otherwise parameter values, including the variance of ϵ , are the *same* in the two economies.

a) In both economies, people's expected value for the inflation rate that will prevail in the distant future is *zero*. How do you know this must be true?

5 pts. The Taylor principle ensures that there is a LRSS inflation rate $\bar{\pi}$ for which:
$\bar{\pi} = \beta\bar{\pi} + \lambda y$ *when y=0.Hence* $\bar{\pi} = 0$.

b) Suppose that you get data from each economy and run regressions. The left-hand side ("dependent") variable in the regressions is inflation π_t . The only right-hand side ("independent") variable in the regressions is the output gap in the same period y_t . Will the estimated coefficient on output be bigger for Mercia, bigger for Wessex, or the same for both countries? Explain. I *do* expect to see a derivation here!

10 pts. Working backward from the LRSS, $\pi_t = E_t[\lambda \sum_{\tau=0}^{\infty} \beta^\tau y_{t+\tau}] = \lambda \sum_{\tau=0}^{\infty} \beta^\tau E_t[y_{t+\tau}]$.

Given the process determining y, $E_t[y_{t+\tau}] = \rho^\tau y_t$.

Hence $\pi_t = \lambda \sum_{\tau=0}^{\infty} \beta^\tau \rho^\tau y_t = \lambda(\beta\rho)^\tau y_t = \dfrac{\lambda}{1-\beta\rho} y_t$

The estimated coefficient on output will be bigger for the economy with a larger value of ρ : bigger for Mercia, smaller for Wessex.

9) Consider a model in which a representative-agent household maximizes

$$E_t \sum_{\tau=0}^{\infty} \beta^\tau \left[\frac{1}{1-\theta}(Z_{t+\tau} C_{t+\tau})^{1-\theta} + \frac{1}{1-\nu}(M_{t+\tau} / P_{t+\tau})^{1-\nu} - \frac{1}{1+\lambda} L_{t+\tau}^{1+\lambda} \right]$$

where C is consumption, *M/P* is real money balance, and L is labor hours, all chosen by the agent. The parameters θ , ν and λ are all between zero and one. Z is a parameter of the felicity function that varies over time in a way the agent cannot control or predict. All that the agent knows is that $Z_t = \bar{Z} + \epsilon_t$ where $\bar{Z}$ is fixed and ϵ is mean-zero i.i.d.

The agent's *nominal* wealth is denoted A . A evolves as

$$A_{t+1} = M_t + (A_t + W_t L_t + R_t u_t \bar{K} - P_t C_t - P_t u_t^\alpha \bar{K} - M_t)(1 + i_t)$$

where i_t is the nominal interest rate. W_t is the nominal wage, determined in a competitive market. P_t is the price level. $\bar{K}$ is a *fixed* quantity of capital owned by the agent; it is exogenous and remains the same in all periods. u_t is the "capital utilization rate," a variable chosen by the agent. R_t is the rental rate for a unit of utilized capital, determined in a competitive market. u_t^α is a real cost of using capital which increases with the utilization rate. Assume that $0 < \alpha < 1$. Assume "certainty equivalence" holds: in the agent's optimization problem, take expected values of future variables to be equivalent to actual known values of future variables.

a) Derive the value of the capital utilization rate u_t that the representative agent will choose, as a function of relevant time-t variables. Holding all other variables fixed, how does an increase in R_t affect the value of u_t chosen by a household - increase, decrease or have no effect on u_t? Compare your answer here with the comparable result in the model of Christiano, Eichenbaum and Evans (2005). *10 pts.*

I said "time-t variables," so intratemporal first order condition:

$$0 = \frac{\partial A_{t+1}}{\partial u_t} = R_t \bar{K} - P_t \alpha u_t^{\alpha-1} \bar{K}$$

$$\text{gives} \quad u_t = \left(\frac{R_t}{P_t} \frac{1}{\alpha} \right)^{-\left(\frac{1}{1-\alpha}\right)}$$

Since $0 < \alpha < 1$, $\partial u_t / \partial R_t < 0$. *This is the opposite of the result in the CEE model.*

b) Starting from the Bellman equation, derive an equation that gives demand for real money balance $(M/P)_t$ as a function of consumption C_t, the nominal interest rate i_t, and any other relevant variables. *5 pts*

See notes. Difference here is that $\frac{\partial V}{\partial C_t} = Z_t^{1-\theta} C_t^{-\theta} - (1+i_t) P_t \frac{\partial E_t V_{t+1}}{\partial A_{t+1}}$

Using that along with $\frac{\partial V}{\partial (M/P)_t} = \ldots\ldots$

$$\text{gives} \quad \frac{M}{P} = \left(\frac{i_t}{1+i_t} \right)^{-1/\nu} Z_t^{-\left(\frac{1-\theta}{\nu}\right)} C_t^{\theta/\nu}$$

Note that $Z \uparrow \rightarrow (M/P)^D \uparrow$, so $\partial (M/P)_t^D / \partial \varepsilon_t > 0$

c) Starting from the Bellman equation, derive an equation that gives consumption C_t as a function of $E_t C_{t+1}$, the "real interest rate" $r_t = i_t - E_t \pi_{t+1}$ (using the usual approximations), and any other relevant variables. *5 pts*

See notes, Difference here is

$$0 = Z_t^{(1-\theta)} C_t^{-\theta} - (1+i_t) P \frac{1}{P_{t+1}} \bar{Z}^{(1-\theta)} E_t C_{t+1}^{-\theta} \beta$$

Note $E_t Z_{t+1} = \bar{Z}$

$$\text{gives} \quad C_t = (Z_t / \bar{Z})^{\frac{1-\theta}{\theta}} (1+r_t)^{-1/\theta} E_t C_{t+1}^{-\theta} \beta^{-1/\theta}$$

d) Now assume:
- Total output is equal to consumption (closed economy, no investment or government purchases).
- the central bank holds the money supply fixed.
- in every period t, $E_t C_{t+1}$ is equal to the long-run steady-state value $\overline{C}$, and the price level is fixed so $E_t \pi_{t+1}$ is always equal to zero (don't ask why).

What relationship would you expect to observe between the realized value of ϵ in a period and the nominal interest rate i in that period? That is, will a high realized value of ϵ_t tend to increase, decrease or have no effect on i_t? Or do you need more information to say? Explain.

10 pts.

The assumptions here imply:

$$Y_t = (Z_t/\overline{Z})^{\frac{1-\theta}{\theta}} (1+i_t)^{-1/\theta}\, \overline{C}^{-\theta}\, \beta^{-1/\theta} \quad \text{while} \quad (\overline{M}/\overline{P}) = \left(\frac{i_t}{1+i_t}\right)^{-1/\nu} Z_t^{-\left(\frac{1-\theta}{\nu}\right)}\, Y_t^{\theta/\nu}$$

This corresponds to the IS/LM model. A high realized value of ϵ_t shifts the IS curve out/up. It shifts the LM curve back/down. So far the effect on i_t is ambiguous. You have to do the math, which shows that the effect on i_t necessarily nets out to zero.

To simplify notation set $\overline{P} = 0$, $\overline{Z} = 1$

$$\overline{M} = i_t^{-1/\nu} (1+i_t)^{1/\nu} Z_t^{-\left(\frac{1-\theta}{\nu}\right)} \left[Z_t^{\frac{1-\theta}{\theta}} (1+i_t)^{-1/\theta} \beta^{-1/\theta} \right]^{\theta/\nu}$$

$$\overline{M} = i_t^{-1/\nu} (1+i_t)^{1/\nu} (1+i_t)^{-1/\theta} Z_t^{-\left(\frac{1-\theta}{\nu}\right)} Z_t^{\frac{1-\theta}{\nu}} \beta^{-1/\nu}$$

$$\overline{M} = i_t^{-1/\nu} \beta^{-1/\nu}$$

$$i_t = \overline{M}^{-\nu} \beta^{-\nu}$$