ANSWERS Hanes's comp questions June 2018

3) Consider a model in which a representative-agent household maximizes:

$$E_t \sum_{\tau=0}^{\infty} e^{-\delta\tau}\left[\ln C_\tau + \frac{1}{1-\nu}\left(\frac{M}{P}\right)_\tau^{1-\nu} - \frac{1}{2}L_\tau^{\,2}\right] \text{ subject to } Z_{t+1} = \frac{P_t}{P_{t+1}}\left[\frac{M_t}{P_t} + (1+i_t)\left(Z_t - \frac{M_t}{P_t} - C_t + (W/P)_t L_t\right)\right]$$

C is the household's real consumption. M is the household's nominal money balance. P is the price level. L is the quantity of labor supplied by the household. δ is the household's subjective rate of time discount. And $0 < \nu < 1$.
In the budget constraint, Z is a household's real wealth. i_t is the nominal interest rate paid on nonmonetary assets. The nominal wage is W_t .

The production function is: $Y_t = K_t^{\alpha} L_t^{1-\alpha}$. The capital stock K is *fixed,* always equal to *one* (1).
There is no government.
Normalize the number of households to one, so that output Y is equal to output per household.

In each period, the household takes as given its wealth entering the period and the price level. Assume "certainty equivalence" holds, so that in the agent's optimization problem you take expected values of future variables to be equivalent to actual known values of future variables. In forming expected values, the household has rational expectations.

a) Starting from the value function, derive an equation that gives the log of consumption c_t (the lower-case letter denotes the log) in terms of $E_t c_{t+1}$ and the real interest rate r_t , using the usual approximations (assume inflation and *i* are sufficiently small). *5 pts. Note that δ is the rate of time-discount. Using the usual methods gives you:*

$$\frac{1}{C_t} = e^{-\delta}\frac{1}{E_t C_{t+1}}\frac{P_t}{P_{t+1}}(1+i_t), \quad C_t = e^{\delta} E_t C_{t+1}\frac{1+\pi_t}{1+i_t} \doteq e^{\delta} E_t C_{t+1}\frac{1}{1+r_t}$$

Then take logs. By "the usual approximations (assume inflation and i are sufficiently small)," I am telling you that r is small so that $\ln(1+r) \approx r$ *. Thus:*

$$c_t = \delta + E_t c_{t+1} - r_t$$

b) Let $\bar{r}$ denote the long-run steady-state real interest rate. What is $\bar{r}$ in terms of the model's parameters? *5 pts.*
There is no productivity growth here so in LRSS $c_t = c_{t+1} = \bar{c}$ *. Hence:*

$$\bar{c} = \delta + \bar{c} - \bar{r}$$

so $\bar{r} = \delta$ *. Now you can see why I used* $E_t \sum_{\tau=0}^{\infty} e^{-\delta\tau}[.....]$ *rather than* $E_t \sum_{\tau=0}^{\infty} \beta^{\tau}[.....]$ *: because in the latter notation* $\bar{r} = -ln(\beta)$ *which is not as neat.*

c) The "real interest rate gap" is $(r-\bar{r})_t$. The "output gap" is $(y-\bar{y})_t$ where $\bar{y}$ denotes long-run steady-state log output. Derive an equation that gives the current output gap in terms of the expected output gap and the current real interest rate gap. *5 pts. Given that there is no investment or government y = c. Hence:*

$$y_t = \delta + y_{t+1} - r_t \quad (\bar{r})$$

$$y_t = y_{t+1} - r_t + \bar{r} = y_{t+1} - (r-\bar{r})_t$$

$$y_t - \bar{y} = y_{t+1} - \bar{y} - (r-\bar{r})_t$$

d) Suppose the real interest rate gap is AR(1) so that $(r-\overline{r})_t = \rho(r-\overline{r})_{t-1} + \epsilon_t$ where ϵ is mean-zero i.i.d. Write an equation that gives the relationship between the current output gap and the current real interest rate gap. *5 pts. Working back from the LRSS where* $(r-\overline{r}) = (y-\overline{y}) = 0$,

$$(y-\overline{y})_t = E_t \sum_{\tau=0}^{\infty} -(r-\overline{r})_{t+\tau} = E_t \sum_{\tau=0}^{\infty} -\rho^{\tau}(r-\overline{r})_t = -\frac{1}{1-\rho}(r-\overline{r})_t$$

e) Suppose the labor market is perfectly competitive and there is no nominal wage rigidity. The household takes the nominal wage *W* as given and chooses the utility-maximizing quantity of labor to supply L^S . *5 pts.*

i) Derive an equation that shows the relationship between the log real wage $(w-p)_t$ and log output y_t .

Working with the value function or going straight to the "intratemporal f.o.c." (the real wage times the marginal utility of consumption equals the marginal disutility of labor) gives you:

$$\left(\frac{W}{P}\right)_t \frac{1}{C_t} = -(-L_t^{\phi}) = L_t^{\phi}$$

We know that $Y_t = C_t$. *From the production function, with K = 1, we have* $L = Y^{\frac{1}{1-\alpha}}$, *So:*

$$\left(\frac{W}{P}\right)_t \frac{1}{Y_t} = Y_t^{\frac{1}{1-\alpha}} \quad , \quad \left(\frac{W}{P}\right)_t = Y_t Y_t^{\frac{1}{1-\alpha}}$$

Solving for W/P and taking logs gives:

$$(w-p)_t = \frac{2-\alpha}{\alpha} Y_t$$

ii) Is the real wage procyclical, countercyclical or acyclical? *"Procyclical" means positively correlated with output or the output gap. In the above, see that the coefficient on y is positive. Thus the real wage is procyclical.*

f) Now suppose the labor market is not necessarily competitive, but the *product* market is perfectly competitive and there is no nominal price rigidity. Firms take the wage *W* and the output price *P* as given and act to maximize profit.

i) Derive an equation that shows the relationship between the log real wage $(w-p)_t$ and log output y_t .

5 pts. Since the product market is perfectly competitive and firms take the wage as given you can use the condition that, for a firm maximizing profit, the real product wage is equal to the marginal product of labor (or the nominal wage equals the marginal revenue product of labor). Or you can set up a profit-maximization problem:

$$\pi = P L_t^{1-\alpha} - W L_t$$

$$\frac{\partial \pi}{\partial L} = 0 = \ldots$$

Either way you get $W / P = (1-\alpha)L^{-\alpha}$. *Using* $L = Y^{\frac{1}{1-\alpha}}$ *and taking logs gives:*

$$(w-p)_t = \ln(1-\alpha) - \frac{\alpha}{1-\alpha} Y_t$$

ii) Is the real wage procyclical, countercyclical or acyclical? *The coefficient on y is negative, so countercyclical. Note that the condition in e and the condition in f cannot be used at the same time. Or rather, if you use them at the same time, you determine y and l. Those values are the natural rates out output and employment.*

4) In standard real business cycle models there are exogenous shocks to total factor productivity ("productivity" shocks), and exogenous shocks to government purchases of goods and services. The models imply that the actual business cycles we observe in the world must be caused by productivity shocks, not shocks to government purchases. Explain. I am not looking for equations here, just words. *10 pts. See notes. For full credit, you had to say what happens to output and consumption in response to an exogenous shock to government purchases of goods and services, what happens to output and consumption in response to an exogenous shock to total factor productivity, and what happens in reality (consumption and output are positively correlated; consumption is procyclical).*

5) Consider the model presented by Bernanke, Gertler and Gilchrist ("The Financial Accelerator in a Quantitative Business Cycle Framework").
a) What would happen to aggregate output if an exogenous shock redistributed wealth from entrepreneurs to households, with no immediate change in total wealth? Explain why. I am not looking for equations here, just words. *5 pts. The reduction in entrepreneur's wealth raises the interest rate an entrepreneur has to pay for a loan from a financial intermediary. That raises the required expected return to the entrepreneur's business project, reduces the amount of capital he demands, and reduces investment and output.*

b) Bernanke, Gertler and Gilchrist argue that such a shock corresponds to a phenomenon in real economies. What phenomenon do they have in mind? *5 pts. Debt deflation, that is, when the price level turns out to be lower than expected when debts were taken out, reducing the real wealth of debtors and raising the real wealth of creditors.*

6) Consider "expectations-augmented" Phillips curves where y denotes the output gap.
a) Describe the assumptions of *two* different models that imply $\pi_t = E_{t-1}\pi_t + \alpha y_t$. *5 pts. See notes. The two models are the Fischer model and the Lucas supply function model. The key assumption of the LSF model is that the yeomen barbers can see the price of the good they sell, but not the prices of the goods they buy (the price level) (or the aggregate demand shock itself).*

b) Describe the assumptions of *two* different models that imply $\pi_t = \beta E_t\pi_{t+1} + \alpha y_t$ where $0 < \beta < 1$. *5 pts. The two models are the Calvo model and the Rotemberg model. The key assumption of the Rotemberg model is that the cost of changing a price is an increasing function, with a positive second derivative, of the difference between the existing price and the revised price.*

7) Consider an economy with a New Keynesian Phillips curve and a New Keynesian IS curve. y is the output gap. r is the gap between the real interest rate and the natural rate of interest. The public's expectations are rational. There is a long-run steady state where $y = 0$, $\pi = 0$. The central bank sets r_t to minimize a loss function: $E\left[\frac{1}{2}y_t^2 + \frac{1}{2}\pi_t^2\right]$
In your answers to the questions below, I am not necessarily looking for equations. Good explanations in words are fine.

a) Suppose $y_t = E_t y_{t+1} - s r_t + u_t$ where $u_t = \rho\epsilon_{t-1} + \epsilon_t$ and ϵ is mean-zero i.i.d.
$\pi_t = E_t\pi_{t+1} + \kappa y_t$
When setting r_t the central bank does *not* know what ϵ_t is, but it knows what ϵ_{t-1} was.
Similarly, at the time the public forms $E_t\pi_{t+1}$ and $E_t y_{t+1}$ it does *not* know what ϵ_t is, but it knows what ϵ_{t-1} was.

Note that the shock is not AR(1). For both the public and the central bank, $E_t u_t = \rho\epsilon_{t-1}$ and $E_t u_{t+1} = 0$ for all t. To solve this problem with math,
- conjecture that $E_t y_{t+1} = E_t\pi_{t+1} = 0$. This is plausible because $E_t u_{t+1} = 0$ for the public.
- solve this problem: $\min_{r_t}\left[\frac{1}{2}\left((-s r_t + \rho\epsilon_{t-1})^2 + \text{Var}\,\epsilon\right) + \frac{1}{2}\left((\kappa^2 \text{illegible})\right)\right.$

which gives $r_t = -(1/s)\epsilon_{t-1}$ which means $y_t = \epsilon_t$, $\pi_t = \kappa\epsilon_t$. (Which confirms the conjecture.)

Or you could use intuition based on your experience with similar models we've seen. In those models, the central bank sets the interest rate to completely counteract the effect of foreseeable shocks to the IS equation. From this, you know that r_t *is positively related to* ϵ_{t-1} *;* π_t *and* y_t *will not be affected by* ϵ_{t-1} *(which is a foreseeable shock to the IS equation) but will be positively affected by* ϵ_t *(which is unforeseeable) . Thus* r_t *is uncorrelated with* π_t *or* y_t *but is positively correlated with* π_{t-1} *.* π_t and π_{t-1} π_t *and* π_{t-1} *are uncorrelated: you could say this is true because* ϵ *is uncorrelated across periods, or because* π_t *must be unaffected by a foreseeable component of demand and* π_{t-1} *reflects that foreseeable component. Your asnwers to the three questions below must refer to the math, or clearly indicate the intuituive logic.*

i) What is the sign of the correlation between r_t and π_t (positive, negative or zero)? Explain or show why. *5 pts.*

ii) What is the sign of the correlation between r_t and π_{t-1} (positive, negative or zero)? Explain or show why. *5 pts.*

iii) What is the sign of the correlation between π_t and π_{t-1} ? Explain or show why. *5 pts.*

b) Now suppose $y_t = E_t y_{t+1} - s r_t$

$$\pi_t = E_t \pi_{t+1} + \kappa y_t + u_t \text{ where } u_t = \rho \epsilon_{t-1} + \epsilon_t \quad \text{and } \epsilon \text{ is mean-zero i.i.d.}$$

Again, when setting r_t the central bank does *not* know what ϵ_t is but it knows what ϵ_{t-1} was; forming $E_t \pi_{t+1}$ and $E_t y_{t+1}$ the public does *not* know ϵ_t but it knows ϵ_{t-1}.

i) What is the sign of the correlation between r_t and π_t (positive, negative or zero)? Explain or show why.

ii) What is the sign of the correlation between r_t and π_{t-1} (positive, negative or zero)? Explain or show why.

iii) What is the sign of the correlation between π_t and π_{t-1} ? Explain or show why.

To solve this problem with math,

- again conjecture that $E_t y_{t+1} = E_t \pi_{t+1} = 0$ *. This is plausible because* $E_t u_{t+1} = 0$ *for the public.*

- solve this problem:

$$\min_{r_t} \left\{ \frac{1}{2}(-\pi_t)^2 + \frac{1}{2}\left(\left(-\kappa s r_t + \rho \epsilon_{t-1} \right)^2 + \text{Var}\, \epsilon_t \right) \right.$$

which gives:

$$r_t = \frac{k}{s(1+k^2)} \epsilon_{t-1}, \quad y_t = -\frac{k}{1+k^2} \epsilon_{t-1}, \quad \pi_t = \frac{1}{1+k^2} \epsilon_{t-1} + \epsilon_t$$

(Which confirms the conjecture.) Or you could use intuition based on your experience with similar models we've seen. In those models, in response to a foreseeable AS ("cost-push") shock, the central bank sets the interest rate to partially but not completely counteract the effect on inflation. That is, in response to a foresee able positive AS shock - here, a positive ϵ_{t-1} *- the central bank causes a recession -* $y_t < 0$ *- but not enough of a recession to hold inflation stable. Of course,* π_t *is positively affected by* ϵ_t *;* y_t *is unaffected by* ϵ_t*. From this, you know that* r_t *is positively related to* ϵ_{t-1} *and hence positively related to* π_{t-1}*.* r_t *is positively related to* π_t *because both are positively related to* ϵ_{t-1} *(the central bank only partially counteracts the effect of* ϵ_{t-1} *on* π_t*). And* π_t *and* π_{t-1} *are positively correlated (both are affected by uncorrelated: you could say this is true because* ϵ *is uncorrelated across periods, or because* π_t *must be unaffected by* ϵ_{t-1}*).*

i) What is the sign of the correlation between r_t and π_t (positive, negative or zero)? Explain or show why. *5 pts.*

ii) What is the sign of the correlation between r_t and π_{t-1} (positive, negative or zero)? Explain or show why. *5 pts.*

iii) What is the sign of the correlation between π_t and π_{t-1} ? Explain or show why. *5 pts.*

8) In Romer's textbook static (one-period) model of imperfect competition, each household *i* operates a monopoly firm and supplies labor to a perfectly competitive labor market. A household-firm does not use its own labor in production, but instead hires labor from the perfectly competitive labor market at a market-clearing nominal wage *W* per unit of labor. Each household acts to maximize:

$U_i = C_i - \frac{1}{\gamma} L_i^{\gamma}$ where C_i is a function of the household's consumption of individual goods. Each household-firm's production function is $Y_i = H_i$, where H is the number of labor units the household-firm hires from the labor market. Demand for the good produced by a household-firm is $Y_i^D = (P_i / P)^{-\eta} Y$ where P is the price level, Y is average real income or real GDP per household and $\eta > 1$.

a) Starting from profit maximization and utility maximization, derive the level of output in the "flex price equilibrium," that is the natural rate of output. *10 pts.See notes.*

b) Show that this level of output is "too low," less than the Pareto-optimal level of output a social planner would choose. *10 pts. See notes. Note you could not merely state that optimal output equals one; you had to solve the maximization problem (this is what "show" means).*