Abstract

Financing terms and investment decisions are jointly determined. This interdependence links firms’ asset and liability sides and can lead to short-termism in investment. In our model, asymmetric information frictions increase with the investment horizon, such that financing for long-term projects is relatively expensive or even rationed. In response, firms whose first-best investment opportunities are long-term may distort their investment towards second-best projects of shorter maturities. This worsens financing terms for firms with shorter maturity projects, inducing them to distort their investment as well. In equilibrium, investment is inefficiently short-term. Moreover, equilibrium asset-side adjustments by firms amplify shocks and, while privately optimal, may be socially undesirable.

Keywords: short-termism, asymmetric information, debt maturity, asset maturity, credit rationing
1 Introduction

Financing terms affect investment decisions and investment decisions affect financing terms. This interdependence creates an intimate link between firms’ asset and liability sides. In particular, when financing for long-term projects is relatively expensive, firms may adjust their investment behavior towards shorter-term projects, even when those are less efficient. In fact, one important phenomenon in the recent financial crisis, and over the business cycle in general, is that maturities shorten both on the liability side and the asset side of firms’ balance sheets.\footnote{See, for example, the recent studies by Mian and Santos (2011) and Chen, Xu, and Yang (2012) that document shortening of debt maturities on firms’ liability sides during downturns. Dew-Becker (2011) documents that downturns are also associated with drops in the maturity (or duration) of the investments firms undertake on the asset side of their balance sheets.}

In this paper we develop an integrated equilibrium framework to study how financing frictions that arise on the liability side affect investments on firms’ asset sides, and vice versa. In our model, frictions from asymmetric information are more pronounced at longer horizons. This leads to less attractive financing terms, or even credit rationing, for long-term investment projects. Firms respond by distorting their asset-side investments by searching for alternative projects of potentially shorter maturity, even if those projects are second best. These asset side adjustments by firms are self-reinforcing, leading to a short-termism spiral. Individual firms’ asset-side decisions endogenously determine the magnitude of the asymmetric information friction faced by all firms, which creates an externality. Hence, relative to the solution to the central planner problem, the competitive equilibrium exhibits inefficient short-termism. Moreover, firms’ equilibrium asset side adjustments amplify shocks and, while privately optimal, can be socially undesirable.
The main friction in our model is asymmetric information about the payoffs of firms’ investment projects. Some firms have “good” projects which are risk-free and have positive NPV. Other firms have “bad” projects, which are risk-shifted versions of good projects: they can default over time, but have a higher payoff conditional on success. Overall, bad projects have negative NPV. Firms seek financing from a financial sector that can observe the maturity (or horizon) of an investment project, but cannot distinguish good and bad projects. Since bad firms can always mimic good firms, the only way for firms to attract funding is through pooling contracts. In our model, this contract optimally takes the form of a debt contract and, in order for the financier to break even, the interest rate on this pooling debt contract has to increase with maturity in order to reflect larger asymmetric information frictions at longer horizons, leading to less attractive financing terms for long-term projects. Beyond a certain maturity, the required increase in interest rates causes firms with good projects not to seek financing. With only bad firms left to seek financing, lending breaks down and maturity rationing arises.

Firms whose first-best projects only receive funding at unfavorable terms (or cannot raise financing at all) react by searching for second-best projects of potentially shorter maturities, for which financing is priced more advantageously. The resulting inflow of second-best projects worsens the pool of funded, shorter-maturity projects. This leads to a negative externality that exacerbates rationing by further shortening the maximum maturity that can be funded and worsening the financing terms for firms that can receive funding. Yet, as financing terms worsen and more firms are rationed, this leads to an additional inflow of second-best projects into the funded region. The process repeats and a short-termism spiral emerges (illustrated in Figure 1). With endogenous asset and liability side, the equilibrium
Figure 1: Illustration of the short-termism spiral that emerges from endogenous adjustments on the asset side.

is thus given by a fixed point: firms’ investment decisions respond optimally to financing frictions on the liability side, while financiers take into account these responses when extending financing.

The resulting competitive equilibrium exhibits investment that is inefficiently short-term compared to a situation where financing terms at each maturity are offered by a central planner. This is the case because through their impact on the quality of the pool of firms seeking financing, the asset side adjustments made by individual firms affect the financing terms faced by all firms, leading to externalities. In fact, when this negative externality from the adoption of second-best project in response to rationing is strong enough, the short-termism spiral can even lead to a complete breakdown of financing across all maturities. When financing terms are offered by a planner, this dynamic is mitigated. In particular, we show that a planner would subsidize long-term projects and tax short-term projects, in order to mitigate the excessive short-termism of the competitive equilibrium.
Another implication of our model is that firms’ asset side adjustments in response to asymmetric information can amplify shocks. For example, an increase in the difference in riskiness between good and bad projects can lead to significantly larger reductions in financed maturities and reductions in surplus in a setting where firms can adjust their asset side, relative to the benchmark in which firms’ asset sides are held fixed. We also show that whether firms’ equilibrium asset-side adjustments increase or decrease surplus depends on the severity of the cross-firm externality. At one extreme, when second-best projects are (essentially) as good as first-best projects, the privately optimal decision of rationed firms to seek shorter maturity projects increases surplus. In this case, rationed firms that adopt shorter maturity projects do not impose an externality on financed firms. As a result, the only consequence from their maturity adjustments is an increase in output as some rationed long maturity projects become funded short maturity projects. On the other hand, when second-best projects are worse than firms’ initial, first-best projects, the dilution of the financed pool by firms who change their maturity can lead to a decrease in surplus. While it is privately optimal for each individual firm to adopt a second-best shorter maturity project, the dilution externality on the pool leads to an overall reduction in surplus. This may be the case even when, seen in isolation, the second-best project has positive NPV.

By endogenizing the asset side, our model highlights a potentially important cost of short-term debt that has not received much attention in the literature. While most of the literature on short-term debt follows Diamond and Dybvig (1983) and Diamond (1991) in highlighting early liquidation of (fixed) investment projects as the main cost of short-term debt,\textsuperscript{2} our

\textsuperscript{2}Among the recent papers in this vein are, for example, Brunnermeier and Oehmke (2012) and He and Xiong (2012).
model highlights a complementary channel: Short-term financing may create endogenous short-termism and thus change firms’ investment behavior on the asset side. In this aspect, the closest related papers are von Thadden (1995) and Dewatripont and Maskin (1995). In von Thadden (1995) firms may choose inferior short-term projects for fear of early liquidation at an interim date. In contrast, in our model it is not the fear of early liquidation that leads to short-termism is that financing is unavailable or only available at unattractive terms for long-term project. In addition, the resulting short-termism in von Thadden (1995) is part of a constrained efficient outcome, while in our framework short-termism is constrained inefficient. In Dewatripont and Maskin (1995) firms may also adopt short-term projects for fear of being liquidated. In their framework, multiple pareto-ranked equilibria can emerge, one of which is inefficiently short-term. Another paper that also stresses asset and liability side interactions is Cheng and Milbradt (2012), who develop a model in which a firm trades off liquidation costs arising from liability side frictions (specifically, debt runs) against asset side distortions that arise from managerial risk-shifting. In contrast to all of these paper, which focus on a single firm in isolation, this paper stresses the cross-firm externalities generated by firms’ investment decisions and how those can lead to substantial knock-on effects and further asset side distortions. In this aspect our paper thus complements Eisfeldt (2004) and Kurlat (2010) in highlighting how asymmetric information can amplify the response of equilibrium prices and quantities to shocks. Finally, as a point of departure, our paper builds on the extensive literature on credit rationing.

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3 For additional models of managerial short-termism in the presence of asymmetric information, see also Narayanan (1985) and Stein (1989).

4 Another recent paper that highlights spillover effects among firms is Bebchuk and Goldstein (2011). In their setup, spillovers arise directly in project payoffs (projects become more attractive as more firms invest), while in our framework spillovers arise due to endogenous asymmetric information on the financing side.

5 For a summary of this literature, a good starting point is the discussion in Bolton and Dewatripont
The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 derives the competitive equilibrium of asset-side choices and liability-side funding terms and contrasts the resulting equilibrium in which the asset side is held fixed. Section 4 demonstrates that, relative to the central planner problem, the competitive equilibrium exhibits excessive short-termism. Section 5 discusses the importance of explicitly taking into account firms’ asset-side responses to financing frictions on the liability side, highlighting in particular the role of asset side adjustments in the amplification of shocks and pointing out that those asset side adjustments, while privately optimal, may be socially undesirable. Section 6 concludes. All proofs are in the appendix.

2 Model Setup

There is a continuum of firms, each of which is born with an (initial) investment project of a maturity \( t \), which is drawn uniformly from the interval \([0, T]\). The maturity of a project indicates how long it takes for the project to pay off: A project of maturity \( t \in [0, T] \) generates cash flow only at date \( t \) and no cash flows beforehand (or after). Firms seek financing from a financial sector composed of a continuum competitive, risk-neutral financiers with deep pockets.

While project maturity is commonly observable (such that there is no asymmetric information about when a particular project pays off), there is asymmetric information about

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the quality of projects. Some firms are born with positive NPV projects (“good” projects), while others are born with negative NPV (or “bad”) projects. Whether a project is good or bad is only observable to the firm, but not to financiers. Financiers merely know that out of firms’ initial projects a fraction $\beta$ has positive NPV, while a fraction $1 - \beta$ has negative NPV.

Both good and bad projects cost 1 dollar to set up, but they differ in the payoffs they generate. Good projects are risk-free and positive NPV. Specifically, they pay off a certain amount $e^{rt}R$ at maturity $t$, where $r$ denotes the constant, exogenous risk-free rate. Good projects thus have constant positive NPV of

$$NPV_G = R - 1 > 0. \quad (1)$$

Bad projects, on the other hand, are risky and have negative NPV. More specifically, at each moment during a project’s life, bad firms engage in risk shifting, such that bad projects pay off only with probability $\Delta e^{-\lambda t}$. However, conditional on success, bad projects pay off $e^{(\lambda + r)t}R$, which is more than the payoff from a good project, $e^{rt}R$. Note that in this specification the manager of a bad firm can undertake some amount of risk shifting at each moment in time, which generates a natural link between project maturity and project risk. The parameter $\lambda > 0$ can be interpreted as a constant intensity of default that results from risk shifting, while $\Delta < 1$ is the probability that the projects fails instantaneously after being undertaken. Taken together, bad projects have a constant negative NPV of$^6$

$^6$Our results are robust to a number of variations in these assumptions. For example, it is not necessary to assume that the drift of bad projects exactly compensates for the default intensity $\lambda$. However, this assumption is convenient because it guarantees that the NPV of bad projects is independent of the project maturity. Hence, our results are driven by differences in asymmetric information frictions across different
\[ NPV_B = \Delta R - 1 < 0. \] (2)

The cash flows that are realized at maturity are private information to the firm and are not contractible, in the spirit of Bolton and Scharfstein (1990) and Hart and Moore (1994). Specifically, we assume that at maturity it can only be verified whether or not a project succeeded, but not which exact cash flow has realized. Hence, a successful firm with a bad project, which receives \( e^{(\lambda+r)t} R \) at maturity, can always pretend to have only received \( e^r t R \), and pocket the difference.

While firms are born with a first-best project of some maturity \( t \) drawn uniformly from \([0, T]\), we allow them to adjust their investment decisions in response to financing frictions. If, for example, firms with long-term projects expect financing terms for long-term projects to be unattractive, they can alter their asset side investment decision in response and attempt to invest in a shorter-term project. However, this asset side adjustment comes at a cost: instead of using some of their capacity to improve the nascent project to a quality \( \beta \) project, they instead invest part of their capacity in finding a new project which again has maturity drawn uniformly from \([0, T]\), but only have enough capacity left over to improve this project to a quality \( \alpha \beta \).

We make the following assumptions to make the analysis of firms’ asset side adjustments tractable: After a firm learns the maturity of its first-best project, but before the quality of the project is revealed to the firm, firms can decide to search for a new project with a different maturities as opposed to differences in NPV across different maturities. It is also not necessary to assume that private information is infinitely lived (i.e., good firms remain good forever, bad firms remain bad forever). We have solved a variant of the model where bad firms switch to being good firms over time. This alternative setting shows that our results are robust to a setting where private information is more pronounced at shorter horizons, while long horizons are characterized by uncertainty rather than just private information.
and potentially shorter maturity. To search for a new project, firms effectively play a bandit: If a firm decides to change its project maturity, the firm receives a new project drawn from the original maturity distribution (i.e., uniform on $[0, T]$). However, this new project is second best, in the sense that the new project is of high quality only with probability $\alpha \beta$, where $\alpha \in [0, 1]$. Drawing a new project has no direct cost for firms (i.e., the firm can play the bandit for free), but it has an implicit cost since the firm loses access to its original project.

The potential adjustments on firms’ asset sides change the distribution of average project quality as a function of maturity. This is depicted in Figure 2: The left panel depicts the initial distribution of projects and their quality across the maturity spectrum. Suppose now that all firms that have projects with a maturity of six years or more search for a new project. The resulting distribution of projects and their quality is given in the right panel of the figure. Whereas before redrawing (left panel) all projects are good with probability $\beta$, after redrawing of projects we obtain a step-function (right panel). To the left of the redrawing cutoff, the average project quality is a mixture of qualities $\beta$ and $\alpha \beta$, whereas to the right of the cutoff all projects have an average quality of $\alpha \beta$.

From the financiers’ point of view, firms with bad projects are contractually indistinguishable from firms with good projects. This is because bad firms can always mimic good firms, such that financiers cannot screen out either type. Hence, financing is possible whenever the

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7While the formulation in the formulation in the paper assumes that firms receive a new project from the original maturity distribution $[0, T]$, one could also assume that a firm with an initial project of maturity $t$ can search for a new project on $[0, t]$. In this alternative specification, search for new projects is directed, in the sense that each firm looking for a new project will receive a project of shorter maturity. We provide a brief outline of the solution of this alternative model in the appendix. An advantage of our formulation is that shortening is not a primitive feature of the redrawing technology but arises endogenously in the equilibrium.
financier can break even on a financing contract that, for a given maturity $t$, pools good and bad firms. We assume that financiers compete by simultaneously posting funding schedules contingent on the project maturity $t \in [0, T]$, taking into account firms’ equilibrium asset side decisions and the resulting quality distribution as a function of project maturity. After the schedules of financing terms have been posted, firms fund themselves at the best rate they can find (if funding is available for the maturity of their investment project). Note that financiers have to break even for every funded maturity, since competition rules out cross-subsidization across maturities.

Given our assumption on the verifiability of cash flows, the optimal financial contract that pools good and bad types takes the form of a debt contract.\(^8\) We assume that in the case of default the entrepreneur incurs a private cost which is proportional to the return generated in the default state. This would occur naturally, for example, if the entrepreneur

\(^8\)Note that because we set the low cash flow to zero, debt contracts can, strictly speaking, also be structured as equity contracts. However, if the payoff in the low state was $L > 0$, then a debt contract becomes strictly optimal: To loosen the incentive constraint in the high state it is always optimal to extract as much as possible in the low state.
holds a small equity stake in the project.\textsuperscript{9} In our analysis, we focus on debt contracts that match the maturity of the project. However, we show in the appendix that this is without loss of generality as the results extend to sequences of rollover debt contracts.\textsuperscript{10}

3 Competitive Equilibrium

A Bayesian Nash equilibrium is given by a set of project redrawal decisions by firms and funding terms offered by financiers that satisfy the following conditions:

**Definition 1** A Bayesian Nash equilibrium is given by (i) project redrawal decisions by firms and (ii) funding conditions offered by the financier sector, such that:

1. Given the funding terms offered, firms that are funded at their original maturity have no incentive to redraw the maturity of their projects (incentive compatibility (IC) constraint).

2. Investors break even at each funded maturity (individual rationality (IR) constraint).

We restrict the funding strategies to strategies that do not allow within \( t \) rationing, that is, for any funded maturity \( t \) all firms can obtain unlimited funding at the posted terms.

3.1 Benchmark: Fixed Asset Side

Before we characterize our equilibrium with endogenous asset side, it is instructive to briefly discuss the equilibrium under the assumption of a fixed asset side, i.e., when firms cannot

\textsuperscript{9}This assumption is equivalent to the cost of the loss of collateral in Stiglitz and Weiss (1981).

\textsuperscript{10}Intuitively, we can focus on debt contracts that match the project maturity without loss of generality because in our model, firms are indifferent between financing though a debt contract that matches maturity and any sequence of rollover debt contracts.
adjust their investment and thus have to stick with the investment project and maturity assigned to them at birth. In this case, we can neglect the IC constraint and the equilibrium is solely driven by the IR constraint. Specifically, a project of maturity $t$ can be financed if the present value of the face value of the pooling debt contract satisfies

$$D(t, \beta) \leq R. \quad (3)$$

If the face value of debt were to exceed $R$, firms with good projects would default for sure and would thus not seek financing in the first place, unraveling the pooling equilibrium. Given a pool quality of $\beta$, because capital markets are competitive, for each maturity $t$ the face value of debt is determined by the financiers’ breakeven condition at that maturity:

$$\beta D + (1 - \beta) \Delta e^{-\lambda t} D = 1. \quad (4)$$

This implies a face value of debt of

$$D(t, \beta) = \frac{1}{\beta + (1 - \beta) \Delta e^{-\lambda t}}. \quad (5)$$

Note that this break-even face value is increasing in project maturity, i.e., $D_t(t, \beta) > 0$, which reflects the riskiness of bad projects at longer horizons (even though their NPV remains fixed). This immediately implies that funding is available up to some maximum maturity $T_B$, beyond which no projects are financed, as illustrated in Figure 3.

The main takeaway from this benchmark case with fixed asset side is that, because asymmetric information frictions increase with project maturity, funding terms for longer
Figure 3: **Fixed Asset Side: Face value and maturity.** The figure illustrates the benchmark case with a fixed asset side. For maturities below 5.9 years, the required face value $D(t, \beta)$ lies below $R$, such that a pooling equilibrium exists. Beyond a maturity of 5.9 years, the required face value exceeds $R$, such that good firms drop out and no pooling equilibrium exists. These maturities cannot be funded in equilibrium.

maturity projects are less favorable than those for shorter maturities. Thus, some long-term projects cannot be funded. In the full model with endogenous asset side, this means that firms with long-term investment projects may look for alternative projects, either to gain funding or to gain funding at better terms. In terms of notation, let $T_B$ be the benchmark funding cutoff and let $W_B$ be the aggregate NPV of all financed projects.

### 3.2 Full Model: Endogenous Asset Side

We now solve for the full equilibrium of our model. This involves making sure that both the IC constraint and the IR constraint discussed above are satisfied. In contrast to the model with fixed asset side, the quality of projects at a given maturity is now endogenous, because it depends on firms’ asset side adjustments (as we illustrated in Figure 2).

As we show in the appendix, the unique (pure-strategy) equilibrium takes the form of a cut-off equilibrium, in the sense that firms with initial maturities beyond this cutoff search for new projects, while firms with initial project maturities below this cutoff stick with their initial project. Given the cutoff nature of the equilibrium, denote by $x(T)$ the average pool
quality on $[0,T]$ under the assumption that firms on $(T,T)$ redraw their maturities. For example, in Figure 2, $T = 6$ so $x(6)$ is the mixture of qualities $\alpha$ and $\alpha \beta$ on the interval $[0,6]$. Given our assumption of a uniform distribution, is is straightforward to show that:

$$x(T) = \beta + \frac{\alpha (T - T)}{2T - T} \leq \beta. \quad (6)$$

Note also that when firms on $(T,T)$ redraw their maturities, the average pool quality on $(T,T]$ is simply given by $\hat{x} = \alpha \beta < x(T)$, because the only firms in this interval are ones who, after searching for a second-best project, again ended up on $(T,T]$. 

Let us now consider the IC and IR constraints. To derive the IC constraint, consider a firm’s incentives to change its maturity. Unfunded firms always have an incentive to search for a new project since it is costless and hence a free option. In determining the IC constraint, we can thus concentrate on the incentives of funded firms to redraw their maturities. To determine the incentives of funded firms, we define the expected profit of a firm of type $\theta \in \{\alpha \beta, \beta\}$ at maturity $t$ accepting competitive funding based on an average project quality of $x$ as

$$
\pi(\theta, t, x) = \theta [R - D(t, x)] + (1 - \theta) \Delta [R - e^{-\lambda t} D(t, x)]
\]

$$
= [\theta + (1 - \theta) \Delta] R - \frac{D(t, x)}{D(t, \theta)} \quad (7)$$

Then, the incentive constraint not to redraw at maturity $t$ is given by

$$\pi(\beta, t, x) \geq \frac{1}{T} \int_{[0,T]} \pi(\alpha \beta, s, x) ds \quad (8)$$
This allows us to define the net value to redrawing as

\[ NVR(t, x) \equiv \frac{1}{T} \int_{[0,T]} \pi(\alpha \beta, s, x) ds - \pi(\beta, t, x) \]  

for a given funded set \([0, T]\) and average quality \(x\) on that funded set.

Among funded firms, the incentive to redraw the maturity of the project is largest for the firm with the highest maturity project as \(NVR_t(t, x) > 0\). The intuition for this finding is as follows. Before a firm learns the quality of its project, it knows that it will have a good project with probability \(\beta\). The market, however, treats this firm as being drawn from a pool of \(x(T) < \beta\). A firm of quality \(\beta\) will thus have to pay a financing rate that is higher than the fair rate on a pool of quality \(\beta\). The gap between the required rate and the fair rate for a pool of quality \(\beta\) increases with project maturity \(t\), such that the benefits of searching for a new project increase with the maturity of the firm’s original project.

Hence, in order to check the IC constraint, we can restrict our attention to the firm located at the switching cutoff \(T\). The firm located at \(T\) has no incentive to redraw, whenever given the induced average quality on \([0, T]\) is such that the net value of redrawing \((NVR)\) is (weakly) negative, i.e., when \(NVR(T, x(T)) \leq 0\). We summarize this in the following Lemma.

**Lemma 1** Suppose firms on \((T, T)\) redraw maturities. Then among the firms that have not redrawn their maturity, the firm right at the cutoff \(T\) has the strongest incentive to redraw. The IC constraint can thus be written as \(NVR(T, x(T)) \leq 0\).

Let us denote the set off redrawing cutoffs that satisfy this incentive constraint by the
We now turn to the financiers’ IR constraint. As we saw in our analysis of financing with an exogenous asset side, in order for financiers to break even a pooling debt contract must be able to attract both types of firm, which requires that $D(t, x(T)) \leq R$ must hold for all financed maturities. Because given a fixed average project quality $x$ the face value of debt is increasing in maturity, $D_t(t, x) > 0$, it is sufficient to check this condition at the cutoff $T$. Hence, we can define the set of cutoff strategies $T$ that can be funded under the conjecture that everyone on $[T, T]$ decides to draw a new project as

$$\mathcal{IR} = \left\{ T \in [0, T] : D(T, x(T)) \leq R \right\}. \quad (11)$$

In words, for each $T \in \mathcal{IR}$ we know that the conjectured cutoff $T$ implies a pool quality $x(T)$ such that at the highest funded maturity, the face value of debt does not exceed $R$.\footnote{Note that the set $\mathcal{IR}$ can have holes. For example, while a conjectured cutoff of nine years may be able to receive funding ($9 \in \mathcal{IR}$ so $[0, 9]$ could be funded) a conjectured cutoff of six years may not be in the funded set ($6 \notin \mathcal{IR}$ so $[0, 6]$ will not be funded), while a cutoff of three years may again be fundable ($3 \in \mathcal{IR}$). The reason is that the dilution of the pool that results when more firms redraw may make the cutoff strategy $T = 6$ not fundable, even if $T = 9$ and $T = 3$ are in the funded set. In the appendix we provide a more detailed characterization of the funded set $\mathcal{IR}$.}

It is instructive to see how $D(T, x(T))$ changes as the funding threshold $T$ changes. Writing out the derivative, we have

$$\frac{dD(T, x(T))}{dT} = D(T, x)^2 \left[ \Delta \lambda e^{-\lambda T} \left(1 - x(T)\right) - \left(1 - \Delta e^{-\lambda T}\right) x'(T) \right]. \quad (12)$$
where $x'(T) > 0$ as a higher funding threshold means less second best projects are funded. Thus, cutting back the maximal funded maturity $T$ has two counterveiling effects. The maturity effect leads to a decrease in the required face value because, all else equal, asymmetric information frictions are less severe at shorter maturities. The dilution effect, on the other hand, means that cutting back the maximal funded maturity will lead to more redrawing and thus will lead to a dilution of the average quality project on the funded interval. This will lead to an increase in the face value required to break even.

Taking into account both the IR and IC constraint, we are now in a position to characterize the equilibrium when the asset side is endogenous, in the sense that firms can respond to asymmetric information frictions by adjusting their investment decisions.

**Proposition 1** The unique competitive equilibrium in pure strategies is a funding cutoff strategy of the form $[0, T^*)$ with

$$T^* = \max \{ IR \cap IC \}.$$ (13)

if $IR \cap IC \neq \emptyset$. Otherwise, no funding is provided in equilibrium.

The intuition for this result is as follows: When financiers compete by posting funding schedules, offering a larger coverage also lowers the rate a financier can offer. The reason is that a larger coverage means that less entrepreneurs change their maturities, such that the average project quality the financier is facing increases, allowing for lower interest rates while still breaking even. The financiers compete until they all offer the maximum coverage that still satisfies both the IR and the IC constraints.
Figure 4: An example of an IC driven equilibrium, with $R > D(T_{IC}, x(T_{IC}))$.

If $T^*$ is on the boundary of $IC$ (i.e., $T^* \in \partial IC$), this means that the firms’ IC constraint is binding at the equilibrium cutoff. In this case we refer to the equilibrium as *IC driven*. Figure 4 depicts such an equilibrium. Compared to the benchmark case in Figure 3, we see that the face-value function now depends on the average quality induced by firms’ asset side adjustments. Moreover, the required face value, for $\alpha < 1$, is always discontinuous at the redrawing cutoff $T$: it jumps from $D(t, x(T))$ to $D(t, \alpha \beta)$ as $t$ crosses the cutoff $T$. Note also that in an *IC* driven equilibrium the face value of debt is still below $R$ at the cutoff $t = T$. This means that financiers would be willing to fund beyond $T$ if firms did not redraw the maturity of their projects once the cutoff $T$ is crossed. However, for firms it is privately optimal to redraw beyond $T$ because the possibility of improved funding conditions at shorter maturities outweighs certain funding at relatively bad terms that would be available were they not to redraw.\(^\text{12}\)

If $T^*$ is on the boundary of $IR$ (i.e., $T^* \in \partial IR$), then the financiers’ IR constraint is binding at $T$ and we call the equilibrium *IR driven*. Figure 5 depicts such an equilibrium. Again, compared to Figure 3, the face-value function is discontinuous at the cutoff $T$. In

\(^{12}\)While not shown in Figure 4, it is possible that in the IC driven equilibrium some firms beyond $T_{IC}$ are financed. This is the case when $D(T_{IC}, \alpha \beta) < R$, such that some firms of quality $\alpha \beta$ can be financed. Although theoretically possible, this case does not arise in any of our numerical examples.
contrast to the IC driven equilibrium in Figure 4, the IR driven equilibrium depicted in Figure 5 exhibits a binding IR constraint at $t = T$, i.e., $D(T, x(T)) = R$.

The mechanism behind this equilibrium is as follows: Although financiers compete locally ($t$ by $t$) by offering schedules, they are aware that any deviation strategy of non-zero measure (e.g., funding some range that was previously unfunded) has an impact on the inventives of all firms as it changes the IC constraint for every $t$. Consequently, financiers internalize the effect of their funding decisions on the average project quality, but competition amongst them restricts their strategy choices. Firms, on the other hand, ignore the impact of the individual decision on the aggregate outcome. This difference between firms and financiers is driven by the scale of their impact: each firm can only undertake one infinitesimal project, and thus cannot affect the aggregate. However, even though a financier is equivalently infinitesimal, since we essentially have Bertrand competition even small deviations lead to non-infinitesimal changes in the aggregate allocations.

### 3.3 Complete Unraveling of Funding

One implication of Proposition 1 is that funding markets can completely unravel. This is the case when after taking into account firms’ equilibrium redrawing behavior and financiers’
equilibrium funding adjustments $IC \cap IR = \emptyset$. A sufficient condition for this result is that $IR = \emptyset$, which is the case when, for each conjectured cutoff $T$ the dilution from firms with second-best projects is strong enough, such that $D(T, x(T)) > R$ for all $T$.

More generally, we can establish the following Proposition:

**Proposition 2** $[0, T] \notin IR$ if and only if $\beta < \min \left\{ \frac{2 - \Delta R}{1 + \alpha (1 - \Delta)} R, \frac{2T - T}{T + \alpha (T - T)} \frac{1 - \Delta e^{-\lambda T}}{1 - \Delta e^{-\lambda T}} R \right\}$.

Proposition 2 implies that cutoffs in the set $[0, T]$ do not satisfy the IR constraint when the average quality of firms $\beta$ is sufficiently low. If this restriction on $\beta$ holds for $T = T_c$, then $IR = \emptyset$. More generally, if the above restriction on $\beta$ holds for $T = \sup IC$, then complete unraveling takes place, since the IR constraint can never be satisfied at the same time as the IC constraint. From the condition in the Proposition, we see that funding is more likely to unravel when the dilution through second best projects is strong (low $\alpha$).

## 4 Central Planner Equilibrium

We now contrast the competitive equilibrium derived above with the allocation that would be implemented by a central planner facing the same informational constraints as the financiers. The main difference between the solution to the constrained planner’s problem and the competitive equilibrium is that, in contrast to the competitive financiers, the planner can cross-subsidize across maturities: While the central planner has to break even, he only faces a break-even constraint over the entire funded interval $[0, T_{cp}]$. Competitive financiers, on the other hand, have to break even for every maturity $t$ separately, which rules out cross-subsidization. As we will show, the social planner subsidizes long-term projects. This allows
more projects to be financed and reduces firms’ incentive to search for second-best projects, and thus leads to higher average project quality.

The central planner’s objective is to pick a funding cutoff $T_{cp}$ and a sequence of face values $D_{cp}(t)$ to maximize aggregate NPV subject to three constraints. First, the planner has to break even (on average) over all funded maturities. Second, firms in the funded region must not prefer to change the maturity of their project. Third, the face value on the funded interval cannot exceed $R$, since otherwise good firms would drop out. We have the following proposition that establishes the optimal central planner funding policy:

**Proposition 3** The central planner optimally sets

$$D_{cp}(t|\hat{t}, T_{cp}) = \begin{cases} \frac{C}{\beta + (1-\beta)\Delta e^{-\lambda t}} & t \leq \hat{t} \\ R & t > \hat{t} \end{cases}$$

where $C = R \left[ \beta + (1 - \beta) \Delta e^{-\lambda \hat{t}} \right]$, $T_{cp}$ fulfills the break-even condition of the central planner, and $\hat{t}$ is the point at which face value reaches $R$. If $\hat{t} > T_{cp}$ the maximal face value $R$ is never charged.

Let us discuss the main differences between the planner’s problem and the competitive equilibrium. Effectively, the planner picks the highest funding cutoff $T_{cp}$ that satisfies all three constraints. This is done by picking a schedule of face values $D_{cp}(t)$ that, at each $t \in [0, T_{cp}]$, makes firms just indifferent between retaining their original project and searching for a new project. The exception to this policy arises in cases where the face value that makes firms indifferent between retaining their original project and searching for a new project is larger than $R$, which happens at $\hat{t}$. Beyond $\hat{t}$ the planner sets the face value to $R$, since
otherwise good firms would drop out. Compared to the competitive equilibrium, the planner thus extracts more surplus at short maturities, which allows the planner to extend funding at maturities that cannot be funded in competitive equilibrium.

The solution to the planner’s problem is illustrated in Figures 6 and 7. Figure 6 depicts the case in which the planner makes all funded firms indifferent between staying at their original maturity in redrawing and picks as the funding cutoff the highest $T$ such that he breaks even over all maturities. Figure 7 depicts the case in which the planner makes firms indifferent between staying and redrawing until the required face value reaches $R$. Beyond that point, the planner keeps the face value at $R$ and funds up to the point where he just breaks even over all maturities. In both cases, the planner charges higher face values for lower maturities—and through this cross-subsidization—is able to fund a larger set of maturities than would be funded in competitive equilibrium. In competitive equilibrium such cross-subsidization is ruled out by maturity-by-maturity competition among financiers, a point that is similar to the impossibility of cross-subsidization between types in Rothschild and Stiglitz (1976) or the inability of a bank to cross-subsidize between patient and impatient consumers in the Diamond and Dybvig (1983) model when those consumers have access to the underlying asset markets (Jacklin (1987)).

Contrasting with the competitive equilibrium, we see that it is the inability of the competitive firms to cross-subsidize across maturities that leads to deviations from the central planner solution, as firms do internalize the impact of their funding decision on the average quality.
Figure 6: Social planner solution with $\hat{t} < T_{cp}$: the solid line depicts $D_{cp}$ whereas the dashed line depicts $D_c$.

Figure 7: Social planner solution with $\hat{t} > T_{cp}$: the solid line depicts $D_{cp}$ whereas the dashed line depicts $D_c$. 
5 Discussion

5.1 The importance of asset side adjustments

Our results highlight the importance considering the maturity of firms’ asset side decisions and their interaction with the pricing of funding instruments of different maturities, for example short-term vs. long-term debt. In fact, the endogeneity of the asset side highlights a potentially important cost of short-term debt that has not received much attention in the literature. While most of the literature on short-term debt follows Diamond and Dybvig (1983) and Diamond (1991) in highlighting early liquidation of (fixed) investment projects as the main cost of short-term debt, our model highlights a complementary channel: Short-term financing generates endogenous short-termism on the asset side and thus changes firms’ investment behavior. In this aspect, our paper thus complements von Thadden (1995) and Dewatripont and Maskin (1995) in highlighting a link between short-term financing and short-termism in investment.

5.2 Amplification of shocks through collective short-termism

One important implication from our model is that endogenous asset side adjustments may amplify shocks, compared to the benchmark case without asset side adjustments. Below, we illustrate this amplification by investigating the comparative statics of our equilibrium in response to changes in $\lambda$. Recall that $\lambda$, which parametrizes the riskiness of the bad project, is a proxy for the severity of asymmetric information.

The comparative statics are illustrated in Figure 8. The left panel illustrates the maximum funded maturity as a function of $\lambda$. The dashed line depicts the benchmark case in
which firms cannot adjust their investment decisions, while the solid line depicts the equilibrium maximum funded maturity after privately optimal asset-side adjustments. In the benchmark case, for low $\lambda$ all maturities receive financing. For larger values of $\lambda$, some long maturities cannot be financed. The solid line depicts the full equilibrium with asset side adjustments. Compared to the benchmark case, the maximum maturity already drops earlier, as firms that could get funding at longer maturities find it more attractive to search for second-best projects of potentially shorter maturities. The solid line also illustrates that the maximum funded maturity drops substantially when $\lambda$ increases beyond 0.05. This sharp drop is caused by a switch from an IC-driven equilibrium to an equilibrium driven by the financiers’ IR constraint.

The right panel depicts the percentage change in surplus (aggregate NPV) when that arises from firms asset-side adjustments. Note that in this example, firms’ privately optimal asset side adjustments lead to uniformly lower output, and a much sharper initial decline in output for lower values of $\lambda$. Hence, as a result of maturity adjustments, output in the case where firms can react to asymmetric information by searching for projects of shorter maturity diverges substantially from the case without redrawing as $\lambda$ increases. Part of this reduction in output comes from a reduction in the total number of projects that is financed, as illustrated by the shaded area. However, a substantial additional effect arises because firms’ asset side adjustments reduce the quality of the average project that is financed. Hence, the overall effect on surplus is driven by both a reduction in quantity of investment and a decrease in the quality of the investment projects undertaken.
Figure 8: Equilibrium with (i) exogenous and (ii) endogenous asset side as a function of $\lambda$. The left panel depicts maximum funded maturities in the benchmark model without asset side adjustments (dashed line) and the full model with endogenous asset side (solid line). The right panel depicts the percentage change in total surplus that results from asset side adjustments relative to the case with exogenous asset side $\Delta W/W_B$ (solid line). The shaded area depicts the percentage change in total lending.

5.3 Are firms’ privately optimal maturity adjustments efficient?

In the above examples, privately optimal maturity adjustments by firms are undesirable from a social perspective because they lead to an unambiguous decrease in output. More generally, however, whether firms’ privately optimal maturity adjustments are socially desirable depends on the degree to which second-best projects are inferior to first best projects, which is parametrized by $\alpha$.

Recall that $T^*$ and $T_B$ denote the equilibrium funding cutoffs in the full model and the benchmark model without maturity adjustments, respectively. To evaluate the effect of firms’ privately optimal maturity adjustments on surplus (aggregate NPV), we can then decompose
the change in total surplus $\Delta W$ into two components, a direct effect and a dilution effect:

$$\Delta W = \left[ \left( \frac{T - T_B}{T} \right) \left( \frac{T_B}{T} \right) NPV (\alpha \beta) \right. - \left. \left( \frac{T_B - T^*}{T} \right) \left[ NPV (\beta) - \left( \frac{T^* + T_B - T}{T} \right) NPV (\alpha \beta) \right] \right]$$

(14)

The direct effect measures the effect of firms’ redrawing behavior keeping the funding threshold constant at $T_B$. If the funding threshold is kept fixed, the ability to redraw projects means that firms with initial project maturities on $[T_B, T]$ now have the possibility of finding funding on $[0, T_B]$. This effect is positive whenever second-best projects have positive NPV, i.e., $NPV (\alpha \beta) > 0$. However, we know that the inflow of second-best projects will change the funding threshold $T_B$ to $T^*$ as dilution of the pool of firms changes financiers’ funding decisions and firms’ redrawing decisions. Because of this dilution effect, funding is only extended for the range $[0, T^*]$ and all firms on $[T^*, T]$ end up redrawing. The dilution effect thus summarizes the output loss of the maturities $[T^*, T_B]$ now becoming unfunded and redrawing. Since $\frac{T^* + T_B - T}{T} < 1$, the term in square brackets is always (weakly) positive, and thus the dilution effect always leads to a decrease in surplus.

To see these two effects at work, consider two special cases. First, when $\alpha = 1$, redrawn projects are just as good as firms’ original projects since $NPV (\beta) = NPV (\alpha \beta)$. In this case, firms’ redrawing behavior does not affect the average quality of projects and the funding cutoff remains constant, $T^* = T_B$. Hence, only the direct effect is present, such that as a result of firm redrawing their projects some firms who were unable to receive financing at their original maturity can now finance an equally attractive positive NPV project of shorter
maturity. This results in an unambiguous increase in welfare. In this case, the ability to search for second best projects helps firms circumvent the adverse selection constraint in the funding market without imposing externalities on other firms.

Second, consider the case in which $\alpha$ is such that redrawn projects have zero NPV. This is the case when $\alpha = \frac{1}{\beta (1 - \Delta R)}$. In this case, a firm that manages to obtain funding by redrawing its maturity adds nothing to the aggregate NPV produced. Hence, the direct effect is not present, such that only the indirect effect is at work: through its redrawing decision the firm reduces the average quality of projects, which leads to a reduction in $T^*$, such that some firms that before were funding their original first-best project are now also forced to redraw. Hence, when second-best projects are zero NPV (or worse), privately optimal redrawing decisions are socially undesirable. For values of $\alpha$ that lie in between these two polar cases both the direct and indirect effects are present.

Finally, let us consider what happens when we pick $\alpha \beta$ such that total lending in the economy remains fixed before and after redrawing. This is the case when $\frac{T^*}{T} + \frac{\Delta R - T^*}{T} = \frac{T_B}{T}$. This implies that $T^* < T_B$, such that we must have $\alpha < 1$. But then we know that $NPV(x) < NPV(\beta)$, which means that on net there is an unambiguous welfare loss: the same number of projects are financed but have a lower average quality. We can thus conclude that for firm’s redrawing decisions to raise surplus it is a necessary condition that lending increases compared to the benchmark model and that second-best projects have positive NPV.

We illustrate this in Figure 9, which plots maximum funded maturities (left panel) and the percentage change in surplus that results from firms’ privately optimal asset side adjustments as a function of the dilution parameter $\alpha$. While the maximum funded maturity is constant
in the benchmark case when firms cannot adjust their maturities (dashed line), it decreases as $\alpha$ decreases when firms can adjust their maturities. When $\alpha$ is lower, the dilutive effect of second-best projects is stronger, leading to a reduction in the maximum funded maturity.

In the right panel, the solid line depicts the percentage change in surplus (aggregate NPV) as a result of firms’ redrawing decisions. When there is no or very little quality difference between first- and second-best projects (i.e., $\alpha$ close to 1), output increases when firms can redraw their project maturity. Thus, for high $\alpha$, the direct effect outweighs the dilution effect: While firms’ maturity adjustments reduce the average quality of funded projects, this negative quality effect is initially outweighed by the increase in the number of projects that can attract financing, which is depicted by the shaded area in the graph. In the figure, this is the case as long as $\alpha > .88$. In this region, firms’ privately optimal maturity adjustments are also socially desirable.

Once $\alpha < .88$, privately optimal asset side adjustments by firms reduce surplus. Note that this reduction in surplus occurs even for parameter values in which firms’ asset side adjustments lead to an increase in lending. An increase in lending is thus not a sufficient condition for an increase in surplus. Specifically, for $\alpha \in (.83,.88)$ the figure shows that output decreases compared to the benchmark model, even though total lending increases. Another observation is that overall output decreases even when second-best projects have positive NPV (which in the Figure is the case as long as $\alpha \geq 0.714$, as marked by the vertical black line). Hence, while a second-best project may be positive NPV when seen in isolation, the dilution externality on other projects can imply that overall output decreases.
Figure 9: Equilibrium with (i) exogenous and (ii) endogenous asset side as a function of the dilution parameter $\alpha$. The left panel illustrates the equilibrium maximum funded maturity. The dashed line depicts the maximum maturity in the benchmark model with exogenous asset side, which does not depend on the dilution parameter $\alpha$. The solid line depicts the maximum funded maturity with endogenous asset side, illustrating how dilution through second-best projects reduces the range of financed maturities. The right panel depicts the percentage change in surplus $\Delta W/W_B$ that results from firms’ privately optimal asset side adjustments (solid black line). The shaded area depicts the percentage change in total lending. The vertical line depicts the value of $\alpha$ for which $NPV(\alpha, \beta) = 0$. Privately optimal asset side adjustments can reduce surplus, even in cases where total lending increases and when second best projects have positive NPV.
6 Conclusion

This paper provides a framework to analyze how financing frictions that arise on the liability side affect firms’ investment decisions on the asset side, and vice versa. In our model, frictions from asymmetric information are more pronounced at longer horizons, leading to less attractive financing terms, or even credit rationing, for long-term investment projects. Firms respond by distorting their asset-side investments by searching for alternative projects of potentially shorter maturity, even if those projects are second best. Because individual firms’ asset-side decisions endogenously determine the magnitude of the asymmetric information friction faced by all firms, an externality arises that leads to inefficient short-termism. In addition, firms’ equilibrium asset side adjustments amplify shocks and, while privately optimal, can be socially undesirable. Broadly speaking, our paper highlights the importance of explicitly taking into account the asset side when analyzing the effect of liability side frictions, such as pressure toward short-term financing.
References


A Appendix

A.1 Comparative statics of D and x

Note that

\[ D(t, x) = \frac{1}{x + (1 - x) \Delta e^{-\lambda t}} \]
\[ \frac{\partial D(t, x)}{\partial t} = (1 - x) \Delta \lambda e^{-\lambda t} D(t, x)^2 > 0 \]
\[ \frac{\partial e^{-\lambda t} D(t, x)}{\partial t} = -x \lambda e^{-\lambda t} D(t, x)^2 < 0 \]
\[ \frac{\partial D(t, x)}{\partial x} = -x \lambda e^{-\lambda t} D(t, x)^2 < 0 \]
\[ x'(T) = \frac{\beta (1 - \alpha)}{(2T - T)^2} > 0 \]
\[ x''(T) = \frac{\beta 2T (1 - \alpha)}{(2T - T)^3} > 0 \]

A.2 Proof of general strategies

A.2.1 Preliminaries

Let the set \( I \) be the set of offered funding so that \( t \in I \) implies that \( t \) is funded at some terms \( D(t, x_t) \). Note that due to competition \( D(t, x_t) \) is offered if \( x_t \) is the rationally expected quality at \( t \). Further, we know that for an equilibrium that \( I \) has to be break-even, so we assume here that \( I \) is break-even. Furthermore, we make \( I \) as tight as possible, that is, if there is some \( \hat{t} \in I \) that does not accept the funding terms and redraws then we define \( I \) excluding \( \hat{t} \). Also, we drop all intervals of measure zero from \( I \) (e.g., isolated points).

Next, note that the mass of redrawn projects on the set \([0, t] \) is given by

\[ F(t, I) = \frac{1}{T} \left[ \int_0^T 1_{\{s \in I\}} \frac{t}{T} ds \right] \]

where \( \frac{t}{T} \) is the probability that a redrawn project is in \([0, t] \). The marginal density contributed by redrawing projects at \( t \) is

\[ \frac{\partial F}{\partial t} = \frac{1}{T} \left[ \int_0^T 1_{\{s \in I\}} \frac{1}{T} ds \right] = \frac{1}{T^2} ||I^c|| \]

where \( I^c = [0, T] \setminus I \). We see that \( \frac{\partial F}{\partial t} \) is linear in the length \(||I^c|| = T - ||I||\) but independent of \( t \). This is because the redrawing technology redistributes everyone evenly on \([0, T] \) regardless of their original \( t \). Suppose further that there is some probability \( p \) with which entrepreneurs who redraw are discovered. Then the average quality on \( I \) is constant and is given by

\[ x(t) = \beta \frac{1 + \alpha p T \frac{\partial F}{\partial t}}{1 + p T \frac{\partial F}{\partial t}} \]

Thus, for any strategy profile \( I \) we know that the average quality on \( I \) is \( x \) and the average quality on \( I^c \) is \( \alpha \beta \) and \( x \) is just influenced by the length of \( I \).

Next, suppose the equilibrium offered financial contract \( I \) has funding holes and is based on an average
quality $x$ on the funded region. Define $T \equiv \max \mathcal{I}$. Consider an alternative contract $\mathcal{E}$ which plugs part of a hole from the left by a measure of $\varepsilon = ||\mathcal{E}||$.

Next, note that

$$\frac{D(t,x)}{D(t,\theta)} > 1 \iff \theta > x$$

and also that

$$\frac{\partial}{\partial t} \left[ \frac{D(t,x)}{D(t,\theta)} \right] = -\lambda \Delta (x-\theta) e^{-\lambda t} D(t,x)^2$$

which is positive for $\theta > x$ and negative for $\theta < x$. To summarize, when $\theta > x$ then the ratio is greater than 1 and increasing in $t$.

Next, define

$$\pi(\theta,t,x) = \theta [R - D(t,x)] + (1-\theta) \Delta [R - e^{-\lambda t} D(t,x)]$$

which is the expected profit of a type $\theta$ of maturity $t$ accepting a contract based on average quality $x$. Note that $\theta = \{\alpha\beta, \beta\}$ for our purposes and that

$$\pi_t(\theta,t,x) = \begin{cases} < 0 & \theta = \alpha \beta, \\ > 0 & \theta = \beta, \end{cases}$$

that is, the expected profit increases with maturity when the type is $\alpha \beta$ (that is, after redrawing), and decreases with maturity when the type is $\beta$ (that is, before redrawing).

First, suppose the IC constraint is not binding, i.e. $NVR(T,\mathcal{I}) < 0$. Then there exists a $\varepsilon > 0$ and $\delta = 0$ that support funding. This is because $\varepsilon > 0 = \delta$ implies more funding is offered (and taken) and thus the average funded quality cannot decline.

### A.2.2 Variational approximation

Next, we will use a variational argument to show that we can always find $\delta > 0, \varepsilon > 0$ so that a deviation strategy can be devised. First, note that for providing funding in the hole, the investor is essentially a monopolist. Thus, let $\hat{D} = D(t,\hat{x}_t)$ be the face value offered in the hole. Next, define

$$cst(\mathcal{E}) = \int_{\mathcal{I}} \pi(\alpha\beta, \hat{s}, \hat{x}) \, ds + \int_{\mathcal{E}} \pi(\alpha\beta, \hat{s}, \hat{x}) \, ds = \pi(\beta, T - \delta, x)$$

where $\varepsilon = ||\mathcal{E}||$ and $\delta$ defines the new marginal agent given funding at terms $\hat{x}$ on the additional set $\mathcal{E}$. We know that by the monopolist assumption that we must have

$$\pi(\beta, t, \hat{x}_t) = cst(\mathcal{E}) \iff \hat{D}(t,\mathcal{E}) = \frac{1}{\hat{x}_t + (1-\hat{x}_t) \Delta e^{-\lambda t}} = \frac{[\beta + (1-\beta) \Delta] R - cst(\mathcal{E})}{\beta + (1-\beta) \Delta e^{-\lambda t}}$$

Next, define the auxiliary function

$$H(\theta, x, A) = \int_{\mathcal{A}} \frac{D(t,x)}{D(t,\theta)} \, dt$$

Then, plugging in $\hat{D}(t,\mathcal{E})$ into the definition of $cst(\mathcal{E})$, we have

$$cst(\mathcal{E}) = \int_{\mathcal{I}} \pi(\alpha\beta, s, x) \, ds + ||\mathcal{E}|| [\alpha\beta + (1-\alpha\beta) \Delta] R + \{cst(\mathcal{E}) - [\beta + (1-\beta) \Delta] R\} H(\alpha\beta, \beta, \mathcal{E})$$
Solving for \( \text{cst}(\mathcal{E}) \), we have

\[
\text{cst}(\mathcal{E}) = \int_{\mathcal{E}} \pi(\alpha\beta, s, x) \, ds + \|\mathcal{E}\| \left[ \alpha\beta + (1 - \alpha\beta) \Delta \right] R - \left[ \beta + (1 - \beta) \Delta \right] R \cdot H(\alpha\beta, \beta, \mathcal{E}) \\
\quad \left(1 - H(\alpha\beta, \beta, \mathcal{E})\right)
\]

and we linearly approximate around \( \varepsilon = \|\mathcal{E}\| = 0 \) to get

\[
\text{cst}(\varepsilon) \approx \text{cst}(0) + \varepsilon \cdot \text{cst}'(0)
\]

where

\[
\text{cst}(0) = \int_{\mathcal{E}} \pi(\alpha\beta, s, x) \, ds = \pi(\beta, T, x) = \left[ \beta + (1 - \beta) \Delta \right] R - \frac{D(T, x)}{D(T, \beta)}
\]

and \( \text{cst}'(0) \) is bounded.

Next, the new marginal agent \( T - \delta \) is defined by

\[
\pi(\beta, T - \delta, x) = \left[ \beta + (1 - \beta) \Delta \right] R - \frac{\beta + (1 - \beta) \Delta e^{-\lambda[T - \delta]}}{x + (1 - x) \Delta e^{-\lambda[T - \delta]}} = \text{cst}(\mathcal{E})
\]

We rearrange to get

\[
\begin{align*}
\delta(T - \delta) &= \frac{x \left[ \beta + (1 - \beta) \Delta \right] R - \text{cst}(\mathcal{E})}{\Delta \left(1 - \beta - (1 - x) \left[ \beta + (1 - \beta) \Delta \right] R - \text{cst}(\mathcal{E}) \right)} \\
&\quad \Leftrightarrow \delta(\mathcal{E}) = \frac{1}{\lambda} \log \left[ \frac{x \left[ \beta + (1 - \beta) \Delta \right] R - \text{cst}(\mathcal{E})}{\Delta \left(1 - \beta - (1 - x) \left[ \beta + (1 - \beta) \Delta \right] R - \text{cst}(\mathcal{E}) \right)} \right] + T
\end{align*}
\]

and we note that \( \delta(0) = 0 \) by the definition of \( \delta \) and the fact that \( \pi(\beta, T, x) = \text{cst}(0) \). We write

\[
\delta(\varepsilon) \approx \delta(0) + \varepsilon \cdot \delta'(0) = \varepsilon \cdot \delta'(0)
\]

where \( \delta'(0) \) is bounded.

Next, we will calculate \( \hat{x}_t, t \in \mathcal{E} \), the true average quality on \( \mathcal{E} \). Note that

\[
\hat{x}(\varepsilon) = x \left( \|\mathcal{I}\| + \varepsilon - \delta(\varepsilon) \right) \approx \hat{x}(0) + \varepsilon \cdot \hat{x}'(0)
\]

and where

\[
\begin{align*}
\hat{x}(0) &= x(\mathcal{I}) = x \\
\hat{x}'(0) &= x'(\mathcal{I}) \left[ 1 - \delta'(0) \right] = \beta \frac{\hat{T}(1 - \alpha)}{(2\hat{T} - \|\mathcal{I}\|)} \left[ 1 - \delta'(0) \right]
\end{align*}
\]

Lastly, note that

\[
\hat{x} + (1 - \hat{x}) \Delta e^{-\lambda} \approx \left[ x + (1 - x) \Delta e^{-\lambda} \right] + \varepsilon \hat{x}'(0) \left( 1 - \Delta e^{-\lambda} \right)
\]

### A.2.3 The main proof

The proof now hinges on the implementability of this funding. We will use the following repeatedly:

\[
\pi(\beta, T, x) = \text{cst}(0) \Leftrightarrow \frac{D(T, x)}{D(T, \beta)} = \left[ \beta + (1 - \beta) \Delta \right] R - \text{cst}(0)
\]

by the assumption of indifference at \( T \). For this, we need

\[
R \geq D(t, \hat{x}_t), \forall t \in \mathcal{E} \\
1 \leq \left[ \hat{x} + (1 - \hat{x}) \Delta e^{-\lambda} \right] D(t, \hat{x}_t), \forall t \in \mathcal{E}
\]
Let us first concentrate on the first equation. Note that we can write
\[
R \geq D(t, \hat{x} t) = \frac{[\beta + (1 - \beta) \Delta] R - \text{cst}(E)}{\beta + (1 - \beta) \Delta e^{-\lambda t}} \\
\approx \frac{[\beta + (1 - \beta) \Delta] R - [\text{cst}(0) + \varepsilon \cdot \text{cst}'(0)]}{\beta + (1 - \beta) \Delta e^{-\lambda t}} \\
= \left[ \frac{D(T, x)}{D(T, \beta)} - \varepsilon \cdot \text{cst}'(0) \right] D(t, \beta)
\]
We note that \( D(t, \beta) \) is increasing in \( t \), so we pick \( t = T \) to make the condition least likely to hold. However, we know that since \( D(T, x) < R \) (as we assumed the IC is tight, so the IR is slack except on a measure zero set), we are done.

Second, let us write out
\[
1 \leq [\hat{x} + (1 - \hat{x}) \Delta e^{-\lambda t}] D(t, \hat{x}_t) \\
= \left[ \hat{x} + (1 - \hat{x}) \Delta e^{-\lambda t} \right] \{[\beta + (1 - \beta) \Delta] R - \text{cst}(E) \} D(t, \beta) \\
\approx \left[ \frac{1}{D(t, x)} + \varepsilon \cdot \hat{x}'(0) (1 - \Delta e^{-\lambda t}) \right] \left[ \frac{D(T, x)}{D(T, \beta)} - \varepsilon \cdot \text{cst}'(0) \right] D(t, \beta) \\
= \frac{D(t, \beta)}{D(t, x)} \frac{D(T, x)}{D(T, \beta)} + \varepsilon \left[ \hat{x}'(0) (1 - \Delta e^{-\lambda t}) \frac{D(T, x)}{D(T, \beta)} D(t, \beta) - \frac{D(t, \beta)}{D(t, x)} \text{cst}'(0) \right] + \varepsilon^2...
\]
Ignoring the \( \varepsilon^2 \) term, we know that
\[
\frac{D(t, \beta)}{D(t, x)} \frac{D(T, x)}{D(T, \beta)} \geq 1
\]
and it only holds with equality for \( t = T \) (assuming \( \alpha < 1 \) and thus \( x < \beta \) for \( ||I|| < T \)). For any \( \gamma \) such that \( T - t \geq \gamma \) (such a \( \gamma \) exists since we know that \( T \) is part of an interval that is not of measure zero) we can find a \( \varepsilon \) small enough such that the above equation holds as
\[
\frac{D(T - \gamma, \beta)}{D(T, \beta)} \frac{D(T, x)}{D(T - \gamma, x)} \approx 1 + \gamma (\beta - x) \Delta e^{-\lambda T} D(T, x) D(T, \beta)
\]
for any \( \beta > x \).

A.3 Omitted proofs in the main text

**Proof: Irrelevance of rollover debt contracts.** We now show that a firm with a project of maturity \( T \) cannot finance via rollover at an intermediate date \( t \). At the rollover date, the financier updates his belief on whether the financed firm is good or bad. Conditional on a firm having survived until the rollover date \( t \), the probability that the firm is good is given by
\[
\hat{\beta}(t) = \frac{\beta}{\beta + \Delta (1 - \beta) e^{-\lambda t}}.
\]
This implies that when rolling over its debt from \( t \) to \( T \), the maximum a firm can credibly promise to repay is
\[
X = e^{-r(T-t)} \left[ \hat{\beta}(t) e^{rT} R + \left( 1 - \hat{\beta}(t) \right) e^{-\lambda (T-t)} e^{rT} R \right] \\
= \hat{\beta}(t) R + \left( 1 - \hat{\beta}(t) \right) e^{-\lambda (T-t)} R \\
= \frac{\beta}{\beta + \Delta (1 - \beta) e^{-\lambda t} R} + \frac{\Delta (1 - \beta) e^{-\lambda t} e^{rT}}{\beta + \Delta (1 - \beta) (e^{-\lambda t} e^{rT} - 1) R}
\]
\[
\approx \frac{\beta}{\beta + \Delta (1 - \beta) e^{-\lambda T}} \left[ \beta + \Delta (1 - \beta) e^{-\lambda T} \right]
\]

where we used the fact that the maximum discounted face-value that a firm can credibly commit to is $R$.

From 0 to $T$ then the firm will be able to promise a maximal repayment of $X$ if it stays alive until $t$, which gives the maximum a firm can promise to repay at 0 as

$$
e^{-rt} X \left[ \beta + \Delta (1- \beta) e^{-\lambda t} \right]$$

$$= \frac{R}{\beta + \Delta (1- \beta) e^{-\lambda t}} \left[ \beta + \Delta (1- \beta) e^{-\lambda t} \right]$$

(A.6)

$$= R \left[ \beta + \Delta (1- \beta) e^{-\lambda t} \right]$$

(A.7)

But this is the same amount the firm can raise by financing directly up to date $T$. Hence, if maturity $T$ is rationed, a firm with a project of maturity $T$ can also not obtain financing by using rollover finance.

**Proof of Lemma 1.** Note that for any arbitrary competitive funding profile $I$ that implies an average quality $x$ we know that $\pi_t (\beta, t, x) < 0$ and thus the firm with the highest $t \in I$ has the highest incentives to redraw.

**Proof of Proposition 1.** When financiers compete by posting funding schedules, we make the following two observations that will guarantee uniqueness. First, we note that a lower cutoff $T$ leads to lower quality on the set $[0, T]$, because $x' (T) > 0$. Second, suppose there is an equilibrium with $T < T^*$ and $T \in IR \cap IC$. By the definition of $T^*$, we also know that by $T < T^*$ there has to exist a $T'$ with $T < T' \leq T^*$ and $T' \in IR \cap IC$. A single financier, by offering funding up to $T'$, will capture the whole market $[0, T']$ as he is able to offer a lower face value than $D (t, x (T))$ and still make a (small) profit. This is because $x (T') > x (T)$. By assumption, we have $NVR (0, x (0)) < 0 < NVR (T, x (T))$ so that by continuity of $NVR (T, x (T))$ we know that it crosses from negative to positive territory at least once, so $E$ is not empty. Competition amongst financiers drives the equilibrium funding schedules to extend all the way to $T^* = \max IR \cap IC$.

**Proof of Proposition 2.** Rewrite the set $IR = \{ T \in [0, T] : 1 \leq f (T) \equiv R [x (T) + (1 - x (T)) \Delta e^{-\lambda T}] \}$. First, note that

$$f'' (T) = Re^{-\lambda T} \left[ \Delta \lambda^2 (1 - x (T)) + 2 \Delta \lambda x' (T) + x'' (T) (e^{\lambda T} - \Delta) \right] > 0$$

since $x' (T) > 0, x'' (T) > 0$. Then, if $f (0) < 1 \iff \frac{\beta (1+\alpha)}{2} < \frac{1-\Delta R}{(1-\Delta)R} \equiv \beta$ and $f (T) < 1 \iff \beta < \frac{2T-T}{T+\alpha(T-T)} \frac{1-\Delta e^{-\lambda T} R}{(1-\Delta e^{-\lambda T}) R}$ we know that $f (T') < 1, \forall T' \in (0, T)$

**A.4 Central planner**

First, note the density of projects on $[0, T_{cp}]$ if $T_{cp}$ is the funding cutoff is $\frac{1}{T} + \frac{2T-T}{T} = \frac{2T-T}{T}$. Then, the central planner program $P$ is to pick a sequence of face-values $D_{cp} (t)$ and a funding cut-off $T_{cp}$, to maximize

$$\max_{T, \{ D (t) \}} \frac{2T-T}{T^2} \int_0^T x (T) R + (1 - x (T)) \Delta R dt$$

subject to the following constraints:

$$(BE) : \frac{2T-T}{T^2} \left[ \int_0^T D (t) [x (T) + (1 - x (T)) \Delta e^{-\lambda t}] - 1 dt \right] \geq 0$$

$$(IC) : \frac{1}{T} \left[ \int_0^T \alpha \beta [R - D (t)] + (1 - \alpha \beta) \Delta [R - e^{-\lambda t} D (t)] dt \right] \leq \beta [R - D (t)] + (1 - \beta) \Delta [R - e^{-\lambda t} D (t)]$$

$$(IR) : D (t) \leq R$$

$\forall t \in [0, T]$

**Proposition 4** The central planner optimally sets

$$D_{cp} (t\mid \hat{t}, T_{cp}) = \begin{cases} \frac{\beta + (1-\beta) \Delta e^{-\lambda T}}{R} & t \leq \hat{t} \\ \frac{\beta + (1-\beta) \Delta e^{-\lambda t}}{R} & t > \hat{t} \end{cases}$$
where \( C = R \left[ \beta + (1 - \beta) \Delta e^{-\lambda t} \right] \) and \( T_{cp} \) fulfills the break-even condition (BE). The (IC) and (IR) are never binding at the same time except on a measure zero set: The (IC) constraint is binding for all \( t \in \left[ 0, \min \left\{ \hat{t}, T_{cp} \right\} \right] \), but the (IR) is slack on this set; whereas on \( t \in \left[ \min \left\{ \hat{t}, T_{cp} \right\}, T_{cp} \right] \) the (IC) constraint is slack and the (IR) constraint is binding. Here, \( \hat{t} \) is the point at which face-value reaches \( R \). If \( \hat{t} > T_{cp} \) the maximal rate \( R \) is never charged.

Proof.

We will solve the problem in steps. First, we observe that the objective function is increasing in \( T \) as

\[
\frac{\partial}{\partial T} \left( \frac{2T - T}{T^2} \right) \int_0^T x(T) R + (1 - x(T)) \Delta R = \frac{2(T - T)}{T^2} R \left[ \Delta + x(T) (1 - \Delta) \right] + \frac{2T - T}{T^2} T x'(T) (1 - \Delta) > 0
\]

and thus the central planner will pick the highest \( T \) that is consistent with the constraints.

Second, the (IC) constraint should be binding for any \( t \) at which (IR) is slack. Suppose that there is an interval that contains \( t' \) for which the (IC) constraint is slack and (IR) is also slack. Then, we can easily increase the debt face value \( D(t') \) by a small amount and still have (IC) satisfied for \( t' \). At the same time, we are decreasing the attractiveness of redrawing, i.e., the LHS of the (IC) constraint for all other maturities \( t \), giving us more slack. Finally, such a move, charging a higher face-value for some maturities while not decreasing the face-value anywhere else clearly satisfies the (BE) constraint. We can thus conclude that for all \( t, t' \) for which \( D < R \) we must have the value of the RHS of the (IC) being equal, that is

\[
\beta [R - D(t)] + (1 - \beta) \Delta [R - e^{-\lambda t} D(t)] = \beta [R - D(t')] + (1 - \beta) \Delta [R - e^{-\lambda t'} D(t')]
\]

which implies that

\[
D(t) = C \frac{1}{\beta + (1 - \beta) \Delta e^{-\lambda t}}
\]

for some constant \( C \) for all \( t \) for which (IR) is slack. The constant \( C \) will be determined by the binding break-even condition (BE).

We will now consider two cases: first, (IR) is slack for all \( [0, T] \) so that \( D(T) < R \). In this case, plugging in \( D(t) \) from above gives

\[
C(T) = \frac{T}{\beta + (1 - \beta) \Delta e^{-\lambda t}} \int_0^T x(T) + (1 - x(T)) \Delta e^{-\lambda t} dt
\]

and we also know that since \( D(T) < R \) we must have \( C < C(T) \equiv R \left[ \beta + (1 - \beta) \Delta e^{-\lambda t} \right] \).

Next, suppose that (IR) is binding for \( t \in (\hat{t}, T) \). Why is this a monotone interval? Suppose not. Then there exists a \( t'' < t < t' \) such that the (IR) is binding at \( t'' \), \( t' \) but not at \( t \). Recall the observation that on the set on which (IR) is slack the RHS is constant. Further note that when \( D = R \), the RHS of the (IC) is

\[
(1 - \beta) \Delta R (1 - e^{-\lambda t})
\]

and thus increasing in \( t \). Thus, we know that the RHS at \( t \) has to satisfy

\[
\beta [R - D(t)] + (1 - \beta) \Delta [R - e^{-\lambda t} D(t)] > (1 - \beta) \Delta R (1 - e^{-\lambda t}) > (1 - \beta) \Delta R \left( 1 - e^{-\lambda t''} \right)
\]

which implies that at \( t \) there is slack in the (IC) condition and in the (IR) condition, which we know from above is a contradiction. Thus, we reinterpret \( t \) as the maturity at which the (IR) constraint first starts binding, that is \( D(\hat{t}) = R \). Then we have from the break-even condition

\[
C \int_0^\hat{t} \frac{x + (1 - x) \Delta e^{-\lambda t}}{\beta + (1 - \beta) \Delta e^{-\lambda t}} dt + R \int_\hat{t}^T (1 - x) \Delta e^{-\lambda t} dt = T
\]
and from the definition of \( \hat{t} \leq T \), \( D(\hat{t}) = R \), we have

\[
C = \frac{R}{\beta + (1 - \beta) \Delta e^{-\lambda \hat{t}}}
\]

We can now fully characterize the equilibrium. There will be two cases.

First, suppose that we have \((IR)\) slack everywhere, so that \( \hat{t} > T \). Then this can only be an equilibrium if \((IC)\) is everywhere binding. It should be clear that increasing \( T \) marginally cannot be an equilibrium, as this would have already been summarized via the functions \( C(T) \) and \( \overline{C}(T) \) and the fact that for this conjectured equilibrium we have \( C(T) < \overline{C}(T) \). Thus, increasing face-value a bit will simply lead to an increase in \( C \) as the RHS has to be held constant.

Second, suppose that the maximal \( T \) that allows a constant RHS still results in slackness in the \((IC)\) constraint. Then, an equilibrium can be found in which \( \hat{t} < T \) and the \((IC)\) constraint is slack while the \((IR)\) is tight on \([\hat{t}, T]\) and vice versa on \([0, \hat{t}]\). Thus, we have

\[
D(t, \hat{t}) = \begin{cases} \frac{C}{\beta + (1 - \beta) \Delta e^{-\lambda \hat{t}}} & t \leq \hat{t} \\ R & t > \hat{t} \end{cases}
\]

The straightforward argument here is that \( \hat{t} < T \) leads to a relaxation of the break-even constraint, and now all funded entrepreneurs are charged more. This of course will lead to a lower value of not redrawing, i.e. a lower RHS. First, note that it is sufficient to check the incentives of the entrepreneur with maturity \( \hat{t} \) (as any \( t > \hat{t} \) has higher incentives to stay). Then, we know that a small shift down in \( \hat{t} \) to \( \hat{t} + \varepsilon \) leads to more revenue that can be used to fund more projects, that is from \( T \) to \( T + \delta(\varepsilon) \), and will lead to an increase in average quality from \( x(T) \) to \( x(T + \delta(\varepsilon)) \). As long as there is not a too large increase in the average quality so that no virtuous circle arises that leads to complete funding everywhere, we know that the \( \varepsilon \) deviation leads to an increase in funding while lowering the RHS for the marginal agent \( \hat{t} \). But since \((IC)\) was slack, a small enough \( \varepsilon \) will still satisfy \((IC)\), while increasing the funded maturity set by \( \delta(\varepsilon) \). No such profitable deviation can be constructed once \((IC)\) is binding.

Therefore, we can establish the following algorithm to find the equilibrium \( T_{CP} \). We know that \((IC)\) is always binding in equilibrium for the marginal entrepreneur \( \min \{ \hat{t}, T \} \).

1. Check if the break-even \( C(T_{CP}) \) under the conjecture that \((IR_E)\) is everywhere slack on \([0, T_{CP}]\) is less than \( \overline{C}(T_{CP}) \) and that \((IC)\) is everywhere tight.

2. If \( C(T_{CP}) > \overline{C}(T_{CP}) \) but \((IC)\) is tight at \( T_{CP} \), then we know that \( \hat{t} < T \) in equilibrium. Every cutoff \( \hat{t} \) defines a \( T \) via the budget equation, that is \( T(\hat{t}) \) so that \( t \) is the choice variable. Then, we pick the maximal \( T(\hat{t}) \) that makes the \((IC)\) binding for \( t \in [0, \hat{t}] \). \( T(\hat{t}) \) is defined as the solution to

\[
R \left[ \beta + (1 - \beta) \Delta e^{-\lambda \hat{t}} \right] = C(\hat{t}, T) = \frac{T - R \int_{\hat{t}}^{T} x + (1 - x) \Delta e^{-\lambda t} dt}{\int_{0}^{\hat{t}} x + (1 - x) \Delta e^{-\lambda t} dt}
\]

and of course we only check for \( T(\hat{t}) > \hat{t} \).

We can only point out some things for the social planner. First, we know that \( D(t) = R, \forall t \in [0, T] \) is never optimal, as then the \((IC)\) constraint is never satisfied. Thus, we know that at \( t = 0 \) the \((IR)\) constraint is slack and the social planner is making a profit. This in turn dictates, by monotonicity, that at \( t = T \), the social planner is making a loss, and that this \( T \) necessarily is larger than \( T_{c} \), the competitive equilibrium. If \( D = R \) at \( t = T \), then we know that \( R \left[ x + (1 - x) \Delta e^{-\lambda T} \right] < 1 \). Competition in this model destroys cross-subsidization. ■
A.5 Model 2: Redraw before type revealed on \([0, t]\)

The mass of redrawn projects on the set \([0, t]\) is given by

\[
F(t, I) = \frac{1}{T} \left[ \int_0^t 1_{\{t \not\in I\}} ds + \int_t^T 1_{\{s \not\in I\}} \frac{t}{s} ds \right]
\]

where \(\frac{t}{\text{max}[s, t]}\) is the probability that a redrawn project originating at \(s\) is in \([0, t]\). The marginal density contributed by redrawing projects at \(t\) is

\[
\frac{\partial F}{\partial t} = \frac{1}{T} \left[ 1_{\{s \not\in I\}} - 1_{\{t \not\in I\}} + \int_t^T 1_{\{s \not\in I\}} \frac{1}{s} ds \right]
\]

and we see that \(\frac{\partial F}{\partial t}\) is log-transform of the lengths of the continuous sets that make up \(I\).

Thus, if only a probability \(p\) survive the redrawing, we have

\[
x(t) = \beta \left( 1 + \alpha p \frac{\partial F}{\partial t} \right)
\]

The intuition is that for a given set \(I\), the lowest maturities face the highest dilution as everyone is just adjusting to shorter maturities.

Suppose monotone cutoff strategy \(I = [0, T]\). Then sufficient to concentrate on \(T = \text{max} I\) as

\[
\text{NVR}(t) = \frac{1}{T} \int_0^t \alpha \beta [R - D(s, x)] + (1 - \alpha \beta) \Delta [R - e^{-\lambda s} D(s, x)] ds
\]

\[
- \{\beta [R - D(t, x)] + (1 - \beta) \Delta [R - e^{-\lambda t} D(t, x)]\}
\]

and

\[
\text{NVR}'(t) = -\frac{1}{t^2} \int_0^t ds + \frac{1}{t} \{\alpha \beta [R - D(t, x)] + (1 - \alpha \beta) \Delta [R - e^{-\lambda t} D(t, x)]\}
\]

\[
+ \beta \frac{\partial D(t, x)}{\partial t} + (1 - \beta) \Delta \frac{\partial e^{-\lambda t} D(t, x)}{\partial t}
\]

\[
= \frac{1}{t^2} \int_0^t \alpha \beta [D(s, x) - D(t, x)] + (1 - \alpha \beta) [e^{-\lambda s} D(s, x) - e^{-\lambda t} D(t, x)] ds
\]

\[
+ \beta \frac{\partial D(t, x)}{\partial t} + (1 - \beta) \Delta \frac{\partial e^{-\lambda t} D(t, x)}{\partial t} > 0
\]

as

\[
\alpha \beta [D(s, x) - D(t, x)] + (1 - \alpha \beta) [e^{-\lambda s} D(s, x) - e^{-\lambda t} D(t, x)] = \frac{(e^{t \lambda} - e^{s \lambda}) \{x [1 + \alpha \beta (\Delta - 1)] - \alpha \beta \Delta\}}{[e^{s \lambda} x + \Delta (1 - x)] [e^{t \lambda} x + \Delta (1 - x)]} > 0
\]

\(^{13}\)That is, if \(I = \bigcup_{i=1}^{n} [L_i, U_i]\) where \(L_i < U_i < L_{i+1}\), then

\[
\frac{\partial F}{\partial t} = \frac{1}{T} \sum_{i=j}^{n} \log \left( \frac{U_i}{\text{max}[L_i, t]} \right)
\]

and where \(j = \inf \{\}\).
because

\[
x [1 + \alpha \beta (\Delta - 1)] - \alpha \beta \Delta |_{x=\beta} = \alpha \beta (1 - \alpha \beta) (1 - \Delta) > 0
\]

\[
x [1 + \alpha \beta (\Delta - 1)] - \alpha \beta \Delta |_{x=\alpha \beta} = \beta [1 - \alpha \beta (1 - \Delta) - \alpha \Delta] > 0
\]

and the term is linear in \( x \). The last line is true because

\[
1 - \alpha \beta (1 - \Delta) - \alpha \Delta > 0
\]

\[
\iff \frac{1 - \alpha \beta}{\alpha - \alpha \beta} > 1 > \Delta
\]
<table>
<thead>
<tr>
<th>Variable</th>
<th>Interpretation</th>
<th>Value</th>
<th>Eq.</th>
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<tr>
<td>$[0,T]$</td>
<td>Range of initial project maturities</td>
<td>$[0,10]$</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Initial population share of good projects</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Default intensity of bad projects</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>$\Delta \in \left(\frac{1}{2}, \frac{1}{R}\right)$</td>
<td>Probability that bad project gets off the ground</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Decrease in probability of good project if switching</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>$R \in (1, 2)$</td>
<td>Discounted payoff of good projects</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>$NPV_G$</td>
<td>NPV good project</td>
<td>0.25</td>
<td>(1)</td>
</tr>
<tr>
<td>$NPV_B$</td>
<td>NPV bad project</td>
<td>$-0.25$</td>
<td>(2)</td>
</tr>
<tr>
<td>$D(t, \beta)$</td>
<td>Discounted face value at maturity $t$ and pool quality $\beta$</td>
<td></td>
<td>(5)</td>
</tr>
<tr>
<td>$x(T)$</td>
<td>Share of good projects on no redrawing interval $[0, T]$</td>
<td></td>
<td>(6)</td>
</tr>
<tr>
<td>$\hat{x} = \alpha \beta$</td>
<td>Share of good projects on redrawing interval $[T, T]$</td>
<td>0.49</td>
<td></td>
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<td>$\mathcal{IC}$</td>
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<td></td>
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<tr>
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<td>Set of funded cutoffs strategies: $T \in \mathcal{IR}$ means $[0, T]$ funded</td>
<td></td>
<td>(11)</td>
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</tbody>
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Table 1: Summary of parameters and variables of the model.