Efficient Recapitalization *

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Abstract

We analyze public interventions to alleviate debt overhang among private firms when the
government has limited information and limited resources. We compare the efficiency
of buying equity, purchasing assets, and providing debt guarantees. With symmetric
information, all interventions are equivalent. With asymmetric information between
firms and the government, buying equity dominates the two other interventions. Under
both symmetric and asymmetric information, the government can improve efficiency by
only implementing an intervention if a minimum number of firms participate. Finally,
we solve for the optimal intervention and show how it can be implemented with preferred
stock and warrants.

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It has been well understood since the seminal work of Myers (1977) that debt overhang can lead to under-investment. Firms in financial distress find it difficult to raise capital for new investments because the proceeds from these new investments end up increasing the value of the existing debt instead of the value of equity.

Debt overhang arises because renegotiations are hampered by free-rider problems among dispersed creditors and by contract incompleteness (Bulow and Shoven (1978), Gertner and Scharfstein (1991), and Bhattacharya and Faure-Grimaud (2001)). A large body of empirical research has shown the economic importance of renegotiation costs for firms in financial distress (Gilson, John, and Lang (1990), Asquith, Gertner, and Scharfstein (1994), Hennessy (2004)). Moreover, from a theoretical perspective, one should expect renegotiation to be costly for at least two reasons. First, the covenants that protect debt holders from risk shifting (Jensen and Meckling (1976)) are precisely the ones that can create debt overhang. Second, debt contracts are able to discipline managers only because they are difficult to renegotiate (Hart and Moore (1995)).

Debt overhang at the firm level can generate negative externalities at the aggregate level. In particular, one firm’s decision to forgo profitable investment opportunities (due to debt overhang) reduces payments to households, which can increase household defaults and thus worsen other firms’ debt overhang. If the household sector is sufficiently weak, this mechanism can generate equilibria in which firms do not invest because they expect other firms not to invest. If an economy suffers from such negative externalities, the social costs of debt overhang exceed the private costs, and there might be room for an intervention by the government. By directly providing capital to firms, the government can alleviate debt overhang and possibly improve economic efficiency. This raises the question of whether and how to intervene optimally.

The goal of our paper is to characterize the optimal form of government interventions against debt overhang. In our model firms differ along two dimensions: the quality of their existing assets and their investment opportunities. Asset quality determines the severity of debt overhang and missed investment opportunities generate welfare losses.

A crucial element of our analysis is the information structure at the time when firms decide whether to participate in a government program. We consider two cases. In the symmetric information case, the government and the private sector only know the joint
distribution of investment opportunities and asset quality. In the asymmetric information case, the private sector knows more than the government about quality and opportunities. Mitchell (2001) reviews the evidence from past financial crises and explains why both cases are relevant in practice.¹

The objective of the government is to increase socially valuable investments while minimizing the deadweight losses from raising new taxes. Different interventions lead to different payoff structures for the equity holders, the debt holders and the government, and it is in general difficult to understand how these affect the firms’ incentives to invest. Our strategy is to compare three specific programs (equity injections, asset purchases, and debt guarantees), before solving for the optimal (second best) intervention.

We find three main results. The first result is that the form of intervention is irrelevant under symmetric information. The intuition for this result is the following. The cost to the government equals the transfer to debt holders minus shareholders’ gains from future investment opportunities. Under symmetric information, the government holds equity holders to their reservation utility, and we can show that all interventions reduce debt overhang to the same extent as long as they provide the same amount of financing. Importantly, government programs generate macroeconomic rents for all firms independent of participation because they reduce negative externalities at the aggregate level. The efficient program therefore makes program implementation conditional on full participation, which allows the government to extract the macroeconomic rents from firms.

Our second result is that buying equity dominates the two other interventions under asymmetric information. The intuition for this result is that an efficient program should limit informational rents. Under asymmetric information, firms can take advantage of the program even though they would be able to invest by themselves, and the shareholders of participating firms are typically not held to their reservation utility. Compared to other interventions, buying equity reduces informational rents because firms with good assets and good investment opportunities have to share their surplus with the government. Again, the government can extract macroeconomic rents by requiring a minimum level of participation for program implementation.

¹Note that we do not consider asymmetric information among private investors. The market failure in our model comes from debt overhang, not from adverse selection. For an analysis of optimal interventions in Lemon markets, see Philippon and Skreta (2009).
Our third result is that an optimal intervention can be implemented by injecting capital in exchange for preferred stock and warrants. The preferred stock is bought at a price above market value and provides a subsidy to equity holders to encourage new investment. The warrants’ strike price is set at the firm’s initial book equity value, which allows the government to extract the entire surplus from future investment opportunities.\footnote{We also show that the preferred stock-warrant intervention is equivalent to the optimal intervention in a setting where asset values and investment opportunities are perfectly known to the government. We note that the government cannot simply use observed market prices to implement the intervention because its intervention may in turn affect prices (see Bond, Goldstein, and Prescott (2010) and Bond and Goldstein (2010)).}

We study two extensions of the model. We first show that heterogeneity among assets makes asset purchase programs less attractive because firms choose to sell their worst assets to the government. The second extension deals with deposit insurance. Deposit insurance decreases the cost of intervention because the government is partly reducing its own expected insurance payments, but deposit insurance does not alter our results on the relative efficiency of the different interventions.

This paper relates to the theoretical literature on government bailouts, which focuses mostly on financial institutions. Gorton and Huang (2004) argue that the government can bail out banks in distress because it can provide liquidity more effectively than private investors. Diamond and Rajan (2005) show that bank bailouts can backfire by increasing the demand for liquidity and causing further insolvency. Diamond (2001) emphasizes that governments should only bail out the banks that have specialized knowledge about their borrowers. Aghion, Bolton, and Fries (1999) show that bailouts can be designed so as not to distort ex-ante lending incentives. Bebchuk and Goldstein (2009) study bank bailouts in a model where banks may not lend because of self-fulfilling credit market freezes. Farhi and Tirole (2009) examine bailouts in a setting in which private leverage choices exhibit strategic complementarities due to the monetary policy reaction. Corbett and Mitchell (2000) discuss the importance of reputation in a setting where a bank’s decision to participate in a government intervention is a signal about asset values, and Philippon and Skreta (2009) provide a general analysis of optimal interventions in Lemon markets. Mitchell (2001) analyzes interventions when there is both hidden actions and hidden information. Landier and Ueda (2009) provide an overview of policy options for bank restructuring.

The paper also relates to the empirical literature on bank bailouts. Allen, Chakraborty, ...
and Watanabe (2009) provide evidence consistent with the main predictions of our model: they find that interventions work best when they target equity injections into the banks that have material risks of insolvency. Giannetti and Simonov (2009) find that bank recapitalizations result in positive abnormal returns for the clients of recapitalized banks as predicted by our debt overhang model. Glasserman and Wang (2009) develop a contingent claims framework to estimate market values of securities issued during bank recapitalizations such as preferred stock and warrants.

Three other theoretical papers share our focus on debt overhang. Kocherlakota (2009) analyzes a model where it is the insurance provided by the government that generates debt overhang. He analyzes the optimal form of government intervention and finds an equivalence result similar to our symmetric information equivalence theorem. Our papers differ because we focus on debt overhang generated within the private sector and we consider the problem of endogenous selection into the government’s programs. In Diamond and Rajan (2009) as in our model, debt overhang makes banks unwilling to sell their toxic assets. In effect, refusing to sell risky assets for safe cash is a form of risk shifting. But while we use this initial insight to characterize the general form of government interventions, Diamond and Rajan (2009) study its interactions with trading and liquidity. In their model, the reluctance to sell leads to a collapse in trading which increases the risks of a liquidity crisis. Bhattacharya and Nyborg (2010) examine bank bailouts in a model with debt overhang and heterogeneity in the support of future distributions of bank asset values. Similar to our paper, they analyze the optimal government intervention in their setting. They show that the optimal intervention may consist of a menu of government interventions. We note that menus may also be optimal in our setting.

Our results can shed light on government actions during the financial crisis of 2007-2009. In October 2008, the US government decided to inject capital into the nine largest US banks under the Trouble Asset Relief Program. Attempts to set up an asset purchase program failed and, after various iterations, the intervention was eventually implemented using preferred stock plus warrants. This is qualitatively similar to the optimal intervention derived in our model.

The paper proceeds as follows. Section 1 provides an example of recapitalization, which conveys the main idea of our analysis. Section 2 sets up the formal model. Section 3 solves
for the decentralized equilibrium with and without debt overhang. Section 4 describes the government interventions. Section 5 compares the interventions under symmetric information. Section 6 compares the interventions under asymmetric information. Section 7 discusses optimal interventions. Section 8 describes two extensions to our baseline model. Section 9 discusses the relation of our results to the financial crisis of 2007-2009. Section 10 concludes.

1 Model

We present a general equilibrium model with a financial sector and a household sector. The model has a continuum of households, a continuum of banks, and three dates, $t = 0, 1, 2$. Figure 1 summarizes the timing, technology, and information structure of the model.

Households are ex-ante identical, risk neutral and only care about consumption at date 2. At time 0, they own the equity and debt of the banks and they owe debt due at time 2 (mortgages, consumer loans, etc.). At time 1, they receive an endowment can lend to banks. At time 2, households receive the payoff from ownership of the financial sector and have to pay their debt to the financial sector. They also receive an exogenous labor income that varies across households. If households cannot pay their debt, the bank takes all their income. For simplicity, there is no deadweight loss of default.

1.1 Banks

All banks are identical at $t = 0$, with existing assets financed by equity and long term debt with face value $D$ due at time 2. At time 1, banks become heterogeneous along two dimensions: they learn about the quality of their existing assets and they receive investment opportunities.

The risky long term assets deliver a random payoff $a = A$ or $a = 0$ at time 2. The probability of a good outcome depends on the idiosyncratic quality of the bank’s portfolio and on the aggregate performance of the economy. We capture macroeconomic outcomes by the aggregate payoff $\bar{a}$, and idiosyncratic differences across banks by the random variable $\varepsilon$. At time 1, all private investors learn the realization of $\varepsilon$ for each bank. We define the

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3 We refer to all financial firms as banks and we assume that banks own industrial projects.
probability of a good outcome conditional on the information at time 1 as:

\[ p(\bar{a}, \varepsilon) \equiv \Pr(a = A|\varepsilon, \bar{a}). \]

The variables are defined so that the probability \( p(\varepsilon, \bar{a}) \) is increasing in \( \varepsilon \) and in \( \bar{a} \). Note that \( p \) is also the expected payoff per unit of face value for existing assets of quality \( \varepsilon \) in the aggregate state \( \bar{a} \). The aggregate payoff rate is simply

\[ \bar{\rho}(\bar{a}) \equiv \int_{\varepsilon} p(\bar{a}, \varepsilon) dF_{\varepsilon}(\varepsilon), \]

where \( F_{\varepsilon} \) is the cumulative distribution of asset quality across banks. The variable \( \bar{a} \) is a measure of common performance for all banks’ existing assets (macro risk) and satisfies the accounting constraint:

\[ \bar{\rho}(\bar{a}) A = \bar{a}. \quad (1) \]

Banks also receive new investment opportunities at time 1. All new investments cost the same fixed amount \( x \) at time 1 and deliver income \( v \in [0, V] \) at time 2. The payoff \( v \) is heterogenous across banks and is known at time 1.\(^4\) A bank’s type is therefore defined by \( \varepsilon \), the bank-specific deviation of asset quality from average bank asset quality and \( v \), the quality of its investment opportunities.

Let \( i \) be an indicator for the bank’s investment decision: \( i = 1 \) if the bank invests at time 1, and zero otherwise. The decision to invest depends on the banks’s type and on the aggregate state, so we have \( i(\varepsilon, v, \bar{a}) \). Banks must borrow an amount \( l \) in order to invest. Since we have normalized the banks’ cash balances to zero, the private funding constraint is \( l = x \cdot i \). We will later allow the government to inject cash in the banks to reduce this funding constraint. At time 2 total bank income \( y \) is:

\[ y = a + v \cdot i, \]

There are no direct deadweight losses from bankruptcy. Let \( r \) be the gross interest rate between \( t = 1 \) and \( t = 2 \). Under the usual seniority rules at time 2, we have the following payoffs for long term debt holders, new lenders, and equity holders:

\[ y^D = \min(y, D); \quad y^l = \min(y - y^D, rl); \quad y^e = y - y^D - y^l. \]

\(^4\)There is no risk conditional on time 1 information as in the benchmark debt overhang model (Myers (1977)).
We assume that long term debt is risky. In the low payoff state, banks always default on senior debt. In the high payoff state, banks always repay senior debt.

**Assumption A1 (Risky Debt):** \( V < D < A \).

If \( a = A \), all liabilities are fully repaid \( (y^D = D \text{ and } y^l = rl) \) and equity holders receive \( y^e = y - D - rl \). If \( a = 0 \), under assumption A1, long term debt holders receive all income \( (y^D = y) \) and other investors receive nothing: \( y^l = y^e = 0 \). Figure 2 summarizes the payoffs to investors.

### 1.2 Households

At time 0 all consumers are identical. Each consumer owns the same portfolio of long term debt and equity of banks. They also have various types of loans due to the banks at time 2 with face value \( A \). These loans could be mortgages, auto loans, student loans, credit card debt, or other consumer loans.

At time 1, each consumer receives an identical endowment \( \bar{w}_1 \) and they have access to a storage technology which pays off one unit of time-2 consumption for an investment of one unit of time-1 endowment. Consumers can also lend to banks. Consumers are still identical at time 1 and we consider a symmetric equilibrium where they make the same investment decisions. They lend \( \bar{L} \) to banks and they store \( \bar{w}_1 - \bar{L} \). At time 2 they receive income \( w_2 \) which is heterogeneous and random across households.\(^5\) Let \( \bar{y}^e \), \( \bar{y}^D \), and \( \bar{y}^l \) be the aggregate payments from equity, long term debt, and short term debt. The total income of the household is therefore:

\[
\bar{n}_2 = \bar{w}_1 - \bar{L} + w_2 + \bar{y}^e + \bar{y}^D + \bar{y}^l
\]

The household defaults if and only if \( \bar{n}_2 < A \). There are no direct deadweight losses of default so the bank recovers \( \bar{n}_2 \) in case of default. The aggregate payments (or average payment) from households to banks are therefore:

\[
\bar{a} = \int \min(\bar{n}_2, A) \, dF_{\bar{w}} (w_2) .
\]

\(^5\)The random variable \( w_2 \) captures net non-financial disposable income, for instance labor income minus unavoidable costs (health, food), so \( w_2 \) can be negative.
Note that the mapping from household debt to bank assets satisfies the aggregate resource constraint but leaves room for heterogeneity of banks’ assets quality captured by the parameter $\varepsilon$. This heterogeneity is needed to analyze the consequences of varying quality of assets across banks, as explained earlier. Finally, we need to impose the market clearing conditions. Let $I$ be the set of banks that invest at time 1:

$$I(\bar{a}) = \{(\varepsilon, v) | i(\varepsilon, v, \bar{a}) = 1\}$$

Aggregate investment at time 1 must satisfy $\bar{l} = \bar{x}(I) \equiv x \int_I dF(\varepsilon, v)$ and consumption (or GDP) at time 2 is

$$\bar{c} = \bar{w}_1 + \bar{w}_2 + \int_I (v - x) dF(\varepsilon, v).$$

## 2 Equilibrium

In this section, we analyze the equilibrium of the model.

### 2.1 First best equilibrium

We assume that households have sufficient endowment to finance all positive NPV projects. The time-1 interest rate is then be pinned down by the storage technology, which is normalized to 1.

**Assumption A2** (Excess Savings): $\bar{w}_1 > \bar{x}(1_{(v > x)})$

In the first best equilibrium, banks choose investments at time 1 to maximize firm value $V_1 = E_1[a] + v \cdot i - E_1[y']$ subject to the time 1 budget constraint $l = x \cdot i$, and the break even constraint for new lenders $E_1[y'] = l$. This implies that $V_1 = E_1[a] + (v - x) \cdot i$. Therefore, investment takes place when $v > x$. The unique first best solution is for investment to take place at time 1 if and only if $v > x$, irrespective of the value of $\varepsilon$ and $E_1[\bar{a}]$. The first best equilibrium is unique and first-best consumption is $\bar{c}^{FB} = \bar{w}_1 + \bar{w}_2 + \int_{v>x} (v - x) dF_v(v)$. We can think of the first best as a world in which banks can pledge the PV of new projects to new creditors. Hence, positive NPV projects can always be financed. Figure 3 illustrates the first best.  

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6 Notice the equivalence between maximizing firm value and maximizing equity value with efficient bar-
2.2 Debt overhang equilibrium without intervention

Under debt overhang, we assume that banks maximize equity value $E_1 [y^e | \varepsilon] = E_1 [y - y^D - y^l | \varepsilon]$ taking as given the payoff function $y^D = \min (y, D)$. Recall that the idiosyncratic shock $\varepsilon$ is known at time 1. With probability $p(\tilde{a}, \varepsilon)$ the bank is solvent and repays its creditors, and shareholders receive $A - D + (v - rl) \cdot i$. With probability $1 - p(\tilde{a}, \varepsilon)$ the bank is insolvent, and shareholders get nothing. Using the break even constraint for new lenders that $r = 1/p(\tilde{a}, \varepsilon)$, equity holders solve:

$$\max_i p(\tilde{a}, \varepsilon) \left[ A - D + \left( v - \frac{x}{p(\tilde{a}, \varepsilon)} \right) \cdot i \right].$$

The condition for investment is $p(\tilde{a}, \varepsilon) > x/v$, and the investment domain under debt overhang is therefore:

$$I(\tilde{a}) = \left\{ (\varepsilon, v) \mid p(\tilde{a}, \varepsilon) > \frac{x}{v} \right\}. \quad (5)$$

At time 2, we aggregate across all banks and we have (bank cash-flow statement)

$$\tilde{a} + \int I \int v dF (\varepsilon, v) = \tilde{y}^e + \tilde{y}^D + \tilde{y}^l$$

aggregate bank income

Payment to households

(6)

Using (2) and (6), we can write household income as:

$$n_2 = w_2 + \tilde{w} + \tilde{a} + \int I \int (v - x) dF (\varepsilon, v) \quad (7)$$

The three unknowns are the repayments from households to banks $\tilde{a}$, the investment set $I$, and the income of households $n_2$. The three equilibrium conditions are therefore (3), (5), (7). We solve the model backwards. First, we examine the equilibrium at time 2, when the investment set is given. We then solve for the equilibrium at time 1, when investment is endogenous.

**Equilibrium at time 2**

Let us define

$$K(I) = \tilde{w} + \int I \int (v - x) dF (\varepsilon, v). \quad (8)$$

gaining. We can always write $V_1 = E_1 [y - y^f] = E_1 [y^e + y^D]$. The maximization program for firm value is equivalent to the maximization of equity value $E_1 [y^e]$ as long as we allow renegotiation and transfer payments between equity holders and debt holders at time 1.
Note that $K$ is pre-determined at date 2. Using equation (8), we can write equation (7) as
\[ n_2 = w_2 + \bar{a} + K. \]
Using (3) we obtain the equilibrium condition for $\bar{a}$:
\[ \bar{a} = \int \min (w_2 + \bar{a} + K, A) dF^w (w_2). \]  
\[ (9) \]
We now make a technical assumption to avoid a multiplicity of equivalent equilibria:

**Technical Assumption:** \[ \int \min (w_2 + \bar{w}_1, A) dF^w (w_2) > 0. \]

The following Lemma gives the properties of the aggregate performance of existing assets at time 2:

**Lemma 1** There exists a unique equilibrium $\bar{a}(K)$ at time 2. Moreover, $\bar{a}$ is increasing and concave in $K$.

**Proof.** The slope on the left hand side of equation (9) is 1. The slope on the right hand side is $F^w (\hat{w}_2) \in [0, 1]$ where $\hat{w}_2 = A - \bar{a} - K$ is the income of the marginal household (the differential of the boundary term is zero since the integrated function is continuous). There is therefore at most one solution. Moreover, the technical assumption ensures that the RHS is strictly positive when $\bar{a} = 0$. When $\bar{a} \to \infty$ the RHS goes to $A$, which is finite. Therefore the equilibrium exists and is unique. At the equilibrium, the slope of the RHS must be strictly less than one, so the solution must satisfy $F (\hat{w}_2) < 1$. The comparative statics with respect to $K$ is:
\[ \frac{\partial \bar{a}}{\partial K} = \frac{F (\hat{w}_2)}{1 - F (\hat{w}_2)} > 0 \]
So the function $\bar{a}$ is increasing in $K$. Moreover we have
\[ \frac{\partial \hat{w}_2}{\partial K} = -1 - \frac{\partial \bar{a}}{\partial K} < 0 \]
Since $\hat{w}_2$ is decreasing in $K$, the slope of $\bar{a}$ is decreasing and the function is concave.  

The shape of the function $\bar{a}$ is intuitive because the impact of additional income only increases payment of households in default. Hence, if the share of households in default decreases with income $K$, the impact of additional income $K$ decreases.

**Equilibrium at time 1**
We can now turn to the equilibrium at time 1. We have just seen in equation (9) that \( \bar{a} \) increases with \( K \) at time 2. At time 1, \( K \) depends on the anticipation of \( \bar{a} \) because investment depends on the expected value of current assets through the debt overhang effect. To see this, let us rewrite equation (8) as:

\[
K(\bar{a}) = \bar{w}_1 + \int_{v > x} (v - x) dF(\varepsilon, v),
\]

(10)

The cutoff \( \hat{\varepsilon} \) is defined implicitly by \( p(\bar{a}, \hat{\varepsilon}) v = x \), which implies \( \frac{\partial \hat{\varepsilon}}{\partial \bar{a}} = -\frac{\partial p/\partial \hat{\varepsilon}}{\partial p/\partial \bar{a}} \) and therefore:

\[
\frac{\partial K}{\partial \bar{a}} = \int_{v > x} (v - x) \frac{\partial p f(v, \hat{\varepsilon}(v))}{\partial \bar{a}} \frac{\partial p/\partial \hat{\varepsilon}}{\partial p/\partial \bar{a}} dv.
\]

This last equation shows that \( K \) is increasing in \( \bar{a} \) since all the terms on the right-hand-side are positive. The economic intuition is straightforward. When banks anticipate good performance on their assets, they are less concerned with debt overhang and are more likely to invest. The sensitivity of \( K \) to \( \bar{a} \) depends on the extent of the NPV gap \( v - x \), the elasticity of \( p \) to \( \bar{a} \), and the density evaluated at the boundary of marginal banks (the term \( \partial p/\partial \hat{\varepsilon} \) is simply a normalization given the definition of \( \varepsilon \)).

Figure 4 illustrates the debt overhang equilibrium. The important question here is whether the equilibrium is efficient. The simplest way to answer this question is to see if a pure transfer program can lead to a Pareto improvement.

### 2.3 Debt overhang equilibrium with cash transfers

We study here a simple cash transfer program. The government announces at time 0 that it gives \( m \geq 0 \) to each bank. The government raises the cash by imposing a tax \( m \) on households’ endowments \( \bar{w}_1 \). The deadweight loss from taxation at time 1 is \( \chi m \). Non distorting transfers correspond to the special case where \( \chi = 0 \).

Consider the investment decision for banks. Banks receive cash injection \( m \). It is straightforward to show that if a bank is going to invest, it will first use its cash \( m \), and borrow only \( x - m \). The break even constraint for new lenders remains \( r = 1/p(\bar{a}, \varepsilon) \). If the bank does not invest it can simply keeps \( m \) on its balance sheet. Equity holders therefore

\[^7\text{We can restrict our analysis to the space where } v > x \text{ since from (5) we know that there is no investment outside this range.}\]
maximize:
\[
\max_i p(\bar{a}, \varepsilon) \left[ A - D + i \cdot \left( v - \frac{x - m}{p(\bar{a}, \varepsilon)} \right) + (1 - i) \cdot m \right].
\]

This yields the investment condition \( p(v - m) > x - m \) which defines the investment domain:
\[
I(\bar{a}, m) = \left\{ (\varepsilon, v) \mid p(\bar{a}, \varepsilon) > \frac{x - m}{v - m} \right\}.
\]

Households do not care about transfers because they are residual claimants: what they pay as taxpayers, they receive as bond and equity holders. We therefore only need to modify the definition of \( K \) to include the deadweight losses at time by replacing \( \bar{w}_1 \) by \( \bar{w}_1 - \chi m \) in equation (8). Conditional on \( K \), the equilibrium at time 2 is unchanged and equation (9) gives the same solution \( \bar{a}(K) \). At time 1 we now have:
\[
K(\bar{a}, m) = \bar{w}_1 + \int \int (v - x)f(v, \varepsilon) \, d\varepsilon \, dv - \chi m.
\]

The cutoff \( \bar{v} \) is defined implicitly by \( p(\bar{a}, \bar{v})(v - m) = (x - m) \). The system is therefore described by the increasing and concave function \( \bar{a}(K) \) in (9) which implies \( d\bar{a} = \bar{a}_K dK \) and the function \( K(\bar{a}, m) \) in (12) which implies \( dK = K_\bar{a} d\bar{a} + K_m dm \).

At this point, we need to discuss briefly the issue of multiple equilibria. Without debt overhang, \( K \) would not depend on \( \bar{a} \) and there would be only one equilibrium. With debt overhang, however, there is a positive feedback between investment, the net worth of households, and the performance of outstanding mortgages. We can rule out multiple equilibria when \( \bar{a}_K K_\bar{a} < 1 \). A simple way to ensure unicity is to have enough heterogeneity in the economy (either in labor income, or in asset quality). When the density \( f \) is small, the slope of \( K \) is also small, and the condition \( \bar{a}_K K_\bar{a} < 1 \) is satisfied. Since multiple equilibria are not crucial for the insights of this paper, we proceed under the assumption that the debt overhang equilibrium is unique.

The impact of cash injection \( m \) on average repayment \( \bar{a} \) is
\[
\frac{d\bar{a}}{dm} = \frac{\bar{a}_K K_m}{1 - \bar{a}_K K_\bar{a}},
\]
and from (4), we see that consumption at time 2 satisfies
\[
d\bar{c} = dK(\bar{a}, m) = \frac{K_m}{1 - \bar{a}_K K_\bar{a}} dm.
\]

\[8\text{We are using the standard notations } \bar{a}_K = \frac{\partial \bar{a}}{\partial \bar{a}} \text{ and } K_\bar{a} = \frac{\partial K}{\partial \bar{a}}.\]

\[9\text{In any case, multiple equilibria simply correspond to the limiting case when } \bar{a}_K K_\bar{a} \text{ goes to one, and, as will be seen shortly, they only reinforce the efficiency of government interventions.} \]
From the definition of the cutoff we get \( \frac{\partial \varepsilon}{\partial m} \frac{\partial p}{\partial \varepsilon} = -\frac{(v-x)}{(v-m)^2} \). Differentiating (12) we therefore have:

\[
\frac{\partial K}{\partial m} = \int_{v>x} \frac{(v-x)^2}{(v-m)^2} \frac{\partial p}{\partial \varepsilon} dv - \chi.
\] (13)

The sensitivity of \( K \) to \( m \) increases in the NPV gap \( v - x \) and the density evaluated at the boundary of marginal banks and decreases in deadweight loss of taxation \( \chi \). Importantly, the equilibrium always improves when \( \chi = 0 \).

**Proposition 1** The decentralized equilibrium under debt overhang is inefficient. Non distorting transfers from households to banks at time 1 lead to a Pareto superior outcome.

Figure 5 illustrates the debt overhang equilibrium with cash transfers. If tax revenues can be raised without costs – i.e., if taxes do not create distortions and if tax collection does not require any labor or capital – then these revenues should be used to provide cash to the banks until debt overhang is eliminated. In such a world the issue of efficient recapitalization does not arise, since the government has in effect access to infinite resources.

If government interventions are costly, however, we see from (13) that the benefits of cash transfers are reduced. The overall impact of the cash transfers can even be negative if deadweight losses are large. In such a world, it become critical for the government to minimize the costs of its interventions. This is the issue that we now address.

3 Macroeconomic rents

We consider first interventions at time 0. This allow us to focus on macroeconomic rents and abstract from informational rents. For interventions at time 0, we show that the critical feature is to allow the government to design programs conditional on aggregate participation. On the other hand, the form of the intervention does not matter.

3.1 Government and shareholders

The objective of the government is to maximize the expected utility of the representative agent. All consumers are risk neutral and identical as of \( t = 0 \) and \( t = 1 \). Hence, the
government simply maximizes
\[ \max_{\Gamma} E[\bar{c}(\Gamma)] \]  

(14)

where \( \Gamma \) describes the specific intervention. Let \( \Psi(\Gamma) \) be the expected net transfer from the government to financial firms. We assume that raising taxes is inefficient and leads to a deadweight loss at time 1 equal to \( \chi \Psi(\Gamma) \). The government takes into account this deadweight loss in its maximization program.\(^{10}\)

We place the same constraints on the government as on the private investors. The government can restrict dividend payments to shareholders at time 1 but the government must follow the rule of law and cannot break private contracts. When participation is decided at time 0, we can without loss of generality consider programs where all banks participate. To be concrete, we consider three empirically relevant interventions: asset purchases, equity injections, and debt guarantees.

In an asset purchase program, the government purchases an amount \( Z \) of long term assets at a per unit price of \( q \). If a bank decides to participate, its cash balance increases by \( m = qZ \) and the face value of its assets becomes \( A - Z \). In an equity injection program, the government offers cash \( m \) against a fraction \( \alpha \) of equity returns. In a debt guarantee program the government insures an amount \( S \) of debt newly issued at time 0 for a per unit fee of \( \phi \). The rate on the insured debt is one and the cash balance of the banks becomes \( m = S - \phi S \).

To study efficient interventions it is critical to understand the participation decisions of equity holders. The following value function will prove useful throughout our analysis. Conditional on a cash injection \( m \), the time 0 value of equity value is:

\[ E_0[y_e | a, m] = \bar{p}(\bar{a}) (A - D + m) + \int \int_{I(\bar{a}, m)} (p(\bar{a}, \varepsilon) v - x + (1 - p(\bar{a}, \varepsilon)) m) dF(\varepsilon, v), \]  

(15)

In this equation, one must of course also recognize that in equilibrium \( \bar{a} \) depends on \( m \), as explained earlier. The first term is the expected equity value of long term assets plus the cash injection using the unconditional probability of solvency \( \bar{p}(\bar{a}) \). The second term is the time 0 expected value of new investment opportunities. This value is positive when the

\(^{10}\)In our simplified framework, if the government had access to non distorting tax instruments, it would be trivial to obtain the efficient outcome simply by subsidizing the banks until they reach the first best investment level. In reality, there are always deadweight losses associated with raising taxes and administering government interventions.
bank’s type belongs to the investment set \( I \) defined in Equation (11). Note that cash adds an extra term because the cash spent on investment is not given to debt holders at time 2. The opportunity cost of cash for investment is therefore less than one.

### 3.2 Free participation

In this section we study interventions with free participation.

**Definition 1** An intervention satisfies free participation if the program offered to a bank only depends on that bank’s participation decision.

Let us first study an asset purchase program. Banks sell assets with face value \( Z \) and receive cash \( m = qZ \). It is easy to see that the government does not want to buy assets to the point that default occurs in both states. We can therefore restrict our attention to the case where \( A - Z > D \). After the intervention, the equilibrium takes place as in the previous section. We know that the investment domain in the equilibrium where all the banks participate is \( I(\bar{a}(m), m) \) defined in (11). From the perspective of the government, we can define:

\[
\hat{I}(m) \equiv I(\bar{a}(m), m).
\]

Let \( T = [\varepsilon_{\text{min}}, \varepsilon_{\text{max}}] \times [0, V] \) be the state space. We then have the following Lemma:

**Lemma 2** An asset purchase program \((Z, q)\) with free participation at time 0 implements the investment set \( \hat{I}(m = qZ) \) at the strictly positive cost:

\[
\Psi_0^{\text{free}}(m) \equiv m \int \int \left( 1 - p(\bar{a}(m), \varepsilon) \right) dF(\varepsilon, v) - \int \int \left( p(\bar{a}(m), \varepsilon) v - x \right) dF(\varepsilon, v). \tag{16}
\]

**Proof.** The cost to the government is \( m - \bar{p}(\bar{a})Z \). The participation constraint of banks is \( E_0[y^e|\bar{a}, m] - \bar{p}(\bar{a})Z \geq E_0[y^e|\bar{a}, 0] \). Using (15), we can write a binding constraint as

\[
\bar{p}(\bar{a})Z = \bar{p}(\bar{a})m + m \int \int \left( 1 - p(\bar{a}, \varepsilon) \right) dF(\varepsilon, v) + \int \int \left( p(\bar{a}, \varepsilon) v - x \right) dF(\varepsilon, v)
\]

16
From the definition of \( p(\bar{a}) \) we then get the cost function \( \Psi_0^{\text{free}}(m) \). Finally, both terms on the RHS of (16) are positive. The first is obvious. The second is also positive because \( p(\bar{a}, \varepsilon) v - x \) is negative over the domain \( \hat{I}(m) \setminus I(\bar{a}, 0) \).

The government’s cost under symmetric information has a natural interpretation in terms of the two terms on the right-hand side of equation (16). The first term reflects the transfer of wealth from the government to the debt holders of banks that do not invest: debt value simply increases by \( (1 - \bar{p}) m \) over the domain \( T \setminus \hat{I}(m) \). The second term measures the subsidy needed to induce investment over the expanded domain \( \hat{I}(m) \) compared to the investment domain \( I(\bar{a}, 0) \). We can now compare the different programs.

**Proposition 2.** An asset purchase program \((Z, q)\) is equivalent to a debt guarantee program with \( S = Z \) and \( q = 1 - \phi \). It is also equivalent to an equity injection program \((m, \alpha)\), where \( m = qZ \) and \( q \) and \( \alpha \) are chosen such that at time 0 all banks are indifferent between participating and not participating in the program. All programs implement the same investment set \( \hat{I}(m) \) and have the same expected cost \( \Psi_0^{\text{free}}(m) \).

**Proof.** See Appendix.

The key to this irrelevance theorem is that banks decide whether to participate before they receive information about investment opportunities and asset values. The government thus optimally chooses the program parameters such that banks are indifferent between participating and not participating. The cost to the government is thus independent of whether banks are charged through assets sales, debt guarantee fees, or equity injections.

### 3.3 Conditional participation

We now focus on the participation constraint. So far we assumed that banks can decide whether to participate independently of other banks’ participation decisions. We now allow the government to condition the program offered to a bank on the participation of other banks. We call this a program with conditional participation. In effect, the offer by the government holds only if all banks participate in the program. The key is that if a bank that was supposed to participate decides to drop out, then the program is cancelled for all
banks. It is straightforward to see that the equivalence result of Proposition 2 holds for conditional programs, and we have the following proposition:

Proposition 3 A program with conditional implements the investment set \( \hat{I}(m) \) at cost

\[
\Psi_0^{\text{cond}}(m) = \Psi_0^{\text{free}}(m) - \mathcal{M}(m)
\]

where \( \mathcal{M}(m) \equiv E_0[\gamma|\bar{a}(m),0] - E_0[\gamma|\bar{a}(0),0] \geq 0 \) measures the macroeconomic rents of the program.

Proof. The government offers a program that is implemented only if all the banks opt in. If they do, the equilibrium is \( \bar{a}(m) \). If anyone drops out, the equilibrium is \( \bar{a}(0) \). Let \( E[\gamma] \) be the expected payments to the government. The participation constraint is \( E_0[\gamma|\bar{a}(m),m] - E[\gamma] \geq E_0[\gamma|\bar{a}(0),0] \). By definition, we have \( E_0[\gamma|\bar{a}(0),0] = E_0[\gamma|\bar{a}(m),0] - \mathcal{M}(m) \). The cost to the government is \( mE[\gamma] \). Using a binding participation constraint, we therefore obtain \( \Psi_0^{\text{cond}}(m) = \Psi_0^{\text{free}}(m) - \mathcal{M}(m) \). ■

The program with conditional participation is less costly because the government appropriates the macroeconomic rents created by its intervention. We can use equation (15) to study these rents. Let \( \Delta_\rho(\varepsilon) = p(\bar{a}(qZ),\varepsilon) - p(\bar{a}(0),\varepsilon) \). We then have

\[
\mathcal{M}(qZ) = \bar{\Delta}_\rho(A - D) + \int_{\hat{I}(0)} \int_{\hat{I}(0)} \Delta_\rho(\varepsilon) dF(\varepsilon,v) + \int_{I(a(qZ),0)} \int_{I(\bar{a}(qZ),0)} (p(\bar{a},\varepsilon)v - x) dF(\varepsilon,v).
\]

This expression decomposes the macroeconomic rents to shareholders into three components. The first term is higher repayment rate on assets in place, the second term is the higher expected value of investments that would have been made even without intervention, and the third term is the expected benefit of expanding the equilibrium investment set. Finally, the costs of the minimum participation program can be negative when the macroeconomic rents are large.

We can therefore summarize our results in the following theorem.

Theorem 1 The form of the intervention is irrelevant when there is no asymmetry of information between the government and the banks. The issue is inefficiently low take up
when banks do not internalize the impact of their participation on the health of other banks. The government must use a minimum participation program in order to appropriate the macroeconomic rents generated by its intervention.

4 Informational rents

In this section we consider interventions at time 1, when banks know their types but the government does not. The macroeconomic rents that we have studied in the previous section still exist but we do not need to repeat our analysis. For brevity, we study only programs with free participation and we focus on the consequences of information asymmetry.

4.1 Participation and investment

The objective function of the government is the same as in the previous section and participation decisions are still based on shareholder value. The main difference is that shareholder value is now condition on each bank’s type \((\varepsilon, v)\). Equity value at time 1 with cash injection \(m\):

\[
E_1[y^e|\varepsilon, v, \bar{a}, m] = p(\bar{a}, \varepsilon)(A - D + m) + 1_{(\varepsilon,v)\in I(\bar{a},m)}(p(\bar{a}, \varepsilon)v - x + (1 - p(\bar{a}, \varepsilon))m)
\]

The description of the programs is the same as before, but the government must now take into account the endogenous participation decisions of banks. Assuming a free program, banks opt in if \(E_1[y^e|\bar{a}, \varepsilon, v, \Gamma]\) is greater than \(E_1[y^e|\bar{a}, \varepsilon, v, 0]\). Consider first a bank that would invest even without participating in the government program. For this bank, the net value of participation is:

\[
U_1(\bar{a}, \varepsilon, v; \Gamma) \equiv E_1[y^e|\bar{a}, \varepsilon, v, \Gamma, i = 1] - E_1[y^e|\bar{a}, \varepsilon, v, 0, i = 1]. \tag{17}
\]

It is clear that the government does not want banks to opt in and then fail to invest. We call this the NIP constraint (No Inefficient Participation):

\[
NIP(\bar{a}, \varepsilon, v; \Gamma) \equiv E_1[y^e|\bar{a}, \varepsilon, v, \Gamma, i = 0] - E_1[y^e|\bar{a}, \varepsilon, v, 0, i = 0] \tag{18}
\]

We will always make sure that \(NIP < 0\). The participation decisions for banks that would not invest alone is then:

\[
L_1(\bar{a}, \varepsilon, v; \Gamma) \equiv E_1[y^e|\bar{a}, \varepsilon, v, \Gamma, i = 1] - E_1[y^e|\bar{a}, \varepsilon, v, 0, i = 0]. \tag{19}
\]
Note that $L_1 > 0$ and $NIP < 0$ implies investment conditional on participation. The participation set of any program $\Gamma$ is therefore

$$
\Omega_1 (\bar{a}, \Gamma) = \{ (\varepsilon, v) \mid L_1 (\bar{a}, \varepsilon, v; \Gamma) > 0 \land U_1 (\bar{a}, \varepsilon, v; \Gamma) > 0 \}. \tag{20}
$$

The investment domain under the program is the combination of the investment set $I (\bar{a}, 0)$ (banks that would invest without government intervention) and the participation set $\Omega_1 (\bar{a}, \Gamma)$:

$$
I_1 (\bar{a}, \Gamma) = I (\bar{a}, 0) \cup \Omega_1 (\bar{a}, \Gamma). \tag{21}
$$

Note that the overlap between the two sets, $I (\bar{a}, 0) \cap \Omega_1 (\bar{a}, \Gamma)$, represents opportunistic participation. Opportunistic participation is inefficient, because the government provides a subsidy to banks that would have invested even without the intervention.

### 4.2 Comparison of benchmark interventions

We now compare the relative efficiency of the three benchmark interventions under asymmetric information. We study first the asset purchase program. The upper participation curve (17) is defined by $U_1^a (\bar{a}, \varepsilon, v; Z, q) = (q - p (\bar{a}, \varepsilon)) Z$. Banks participate only if the price $q$ offered by the government the true value $p (\bar{a}, \varepsilon)$ of the assets. This is the adverse selection problem. The NIP constraint (18) only requires $q < 1$, which is always satisfied by efficient interventions. The lower bound schedule (19) is given by $L_1^a (\bar{a}, \varepsilon, v; Z, q) = p (\bar{a}, \varepsilon) v - x + (q - p (\bar{a}, \varepsilon)) Z$. The lower- and the upper-schedules define the participation set $\Omega_1^a (\bar{a}, Z, q)$ from (20). The expected cost of the asset purchase program is:

$$
\Psi_1^a (\bar{a}, q, Z) = Z \int_{\Omega_1^a (\bar{a}, Z, q)} (q - p (\bar{a}, \varepsilon)) dF (\varepsilon, v). \tag{22}
$$

Figure 6 shows the investment and participation sets for asset purchases under asymmetric information. The figure distinguishes three regions of interest: efficient participation, opportunistic participation, and independent investment. The efficient participation region comprises the banks that participate in the intervention and that invest because of the intervention. The opportunistic region comprises the banks that participate in the intervention but would have invested even in the absence of the intervention. The independent investment region comprises the banks that invest without government intervention. As is clear
from the figure, the government’s trade-off is between expanding the efficient participation region and reducing the opportunistic participation region.

From cost equation (22) we see that an asset purchase $qZ$ is less costly than an equivalent free cash injection $qZ$ for three reasons. First, the independent investment region reduces opportunistic participation without reducing investment. Second, the pricing $q < 1$ excludes banks that would not invest. Third, the government receives $Z$ in the high-payoff state which lowers the government’s cost without affecting investment. Let us now compare asset purchases to debt guarantees:

**Proposition 4** *Equivalence of asset purchases and debt guarantees.* An asset purchase program $(Z, q)$ with participation at time 1 is equivalent to a debt guarantee program with $S = Z$ and $q = 1 − \phi$.

**Proof.** See Appendix. □

The equivalence of asset purchases and debt guarantees comes from the fact that both programs make participation contingent on asset quality $p(\bar{a}, \varepsilon)$ but not investment opportunity $v$. To see this result, consider the upper-bound schedule. If $q = 1 − \phi$, banks with asset quality $p \in [1 − \phi, 1]$ choose not to participate. Hence, asset purchase program and debt guarantees have the same upper-bound schedule. Next, note that the net benefit of asset purchases is $(q − p)$, whereas the net benefit of debt guarantees is $(1 − \phi − p)$. Hence, asset purchases and debt guarantees have the same lower bound schedule. The NIP constraint for asset purchases is $p < 1$, which is equivalent to $\phi > 0$. The last step is to show that both asset purchases and debt guarantees have the same cost to the government, which is true since they yield the same net benefit to participants. We can finally compare debt guarantees and asset purchases to equity injections:

**Proposition 5** *Dominance of equity injection.* For any asset purchase program $(Z, q)$ with participation at time 1, there is an equity program that achieves the same allocation at a lower cost for the government.

**Proof.** See Appendix. □
The dominance of equity injection over debt guarantees and asset purchases comes from the fact the equity injections are dependent both on asset quality $\varepsilon$ and investment opportunity $v$. To understand this result, it is helpful to define the function $X(\bar{a}, \varepsilon; m, \alpha)$ as the part of the net benefit from participation that is tied to existing assets:

$$X(\bar{a}, \varepsilon; m, \alpha) \equiv (1 - \alpha) m - \alpha p(\bar{a}, \varepsilon) (A - D).$$

(23)

In words, a participating bank receives net cash injection $(1 - \alpha) m$ and gives up share $\alpha$ of the bank’s expected equity value $p(\bar{a}, \varepsilon) (A - D)$. To compare equity injections with other programs, start by choosing an arbitrary asset purchase program. Then choose $X(\bar{a}, \varepsilon; m, \alpha)$ such that the lower-bound schedule of the asset purchase program coincides with the lower-bound schedule of the equity injection program. Under both programs, equity holders at the lower-bound schedule receive no surplus and are indifferent between participating and not participating.

For given level of asset quality $\varepsilon$, the cost of participation for banks with a good investment opportunity $v$ is higher under the equity injection program than under the asset purchase program because the government receives a share in both existing assets and new investments. As a result, there is less opportunistic participation with equity injections than with asset purchases.

Figure 7 shows the participation and investment regions under the equity injection program. The increase in cost of participation relative to the asset purchase program has two effects. First, conditional on participation, the cost to the government is smaller because the government receives a share in the investment opportunity $v$. Second, there is less opportunistic participation because participation is more costly. As a result, equity injections and asset purchases implement the same level of investment but equity injections are less costly to the government relative to asset purchases.\(^\text{11}\) Finally, the macroeconomic feedback from equation (12) only reinforces the dominance of equity injection.

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\(^{11}\)The final step in the proof is to show that the NIP constraint is the same under both programs. This is true because the equity injection provides lower rents to participating banks than the asset purchase programs. Hence, if the no-efficient participation holds under the asset purchase program, it also holds under the equity injection program. We can also show that equity programs at time 1 cannot be improved by mixing them with a debt guarantee or asset purchase program. Pure equity programs always dominate. The proof is available upon request.
4.3 Optimal interventions

Our third main result is that optimal interventions can be implemented with warrants and preferred stock. To derive the optimal intervention, we first characterize the minimum cost of a government intervention. Under the optimal intervention, the government injects cash \( m \) at time 1 in exchange for state contingent payoffs at time 2. New lenders at time 1 must break even and we can without loss of generality restrict our attention to the case where the government payoffs depend on the residual payoffs \( y - y^D - y' \). As in previous sections, we will examine cost minimization for a given investment set.\(^{12}\) Let us start with a general characterization of the minimum cost for any intervention:

**Lemma 3** Any program with voluntary participation of equity holders over the set \( \Omega \) has a minimum cost of

\[
\Psi_1^{\text{min}} = - \int_{\Omega} (p(\bar{a}, \varepsilon) v - x) dF(\varepsilon, v)
\]

**Proof.** Voluntary participation means that equity holders must get at least \( p(A - D) \). The government and old equity holders must share the residual surplus whose value is

\[
p(A - D) + p(\bar{a}, \varepsilon)v - x
\]

Hence the expected net payments to the government must be

\[
\int_{\Omega} (p(\bar{a}, \varepsilon)v - x) dF(\varepsilon, v)
\]

These payments are negative as long as \( \Omega \) extends the debt overhang investment set.

A simple way to understand this result is to imagine what would happen if the government could observe the types and write contracts contingent on investment. For the shareholders of type \((\varepsilon, v)\), the value of investment is \( p(\bar{a}, \varepsilon)v - x \), which is negative outside the debt overhang investment region \( I(\bar{a}) \). If the government had full information, it could offer a contract with a type-specific payment contingent on investment. The minimum the government would have to offer type \((\varepsilon, v)\) would be \(- (p(\bar{a}, \varepsilon)v - x)\).

\(^{12}\)In general, the government can offer a menu of contracts to the banks in order to obtain various investment sets. The actual choice depends on the distribution of types \( F(p, v) \) and the welfare function \( W \) but we do not need to characterize it. We simply show how to minimize the cost of implementing any particular set.
However, it is far from obvious whether the government can reach this lower bound without full information. The surprising result is that it can do so with warrants and preferred stock.

**Theorem 2** Consider the program $\Gamma = \{m, h, \varepsilon\}$ where the government provides cash $m$ at time 1 in exchange for preferred stock with face value $(1 + h)m$ and a portfolio of $(1 - \eta) / \eta$ warrants at the strike price $A - D$. This program implements the investment set $I_1 (\bar{a}, Z, q)$, where we identify the cash injection by $m = qZ$ and the interest rate on preferred stocks by $q = 1 / (1 - h)$. In the limit $\eta \to 0$, opportunistic participation disappears and the program achieves the minimum cost:

$$\lim_{\eta \to 0} U_1 (\bar{a}, \Gamma) = L_1 (\bar{a}, \Gamma),$$

$$\lim_{\eta \to 0} \Psi_1 (\bar{a}, \Gamma) = \Psi_1^{\text{min}}.$$ 

**Proof.** See Appendix. ■

Figure 8 shows the investment and participation region under the optimal intervention. Under the optimal intervention, the initial shareholders receive the following payoff in the high-payoff state:

$$f (y^e) = \min (y^e, A - D) + \eta \max (y^e - A - D, 0)$$

Shareholders are full residual claimants up to the face value of old assets $A - D$ and $\eta$ residual claimants beyond. When $\eta$ goes to zero, the entire increase in equity value over the no-investment case is extracted by the government via warrants. As a result, the opportunistic participation region disappears and only the banks that really need the capital injection to invest participate in the program.  

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13Note that the government cannot simply overcome asymmetric information by learning about asset values and investment opportunities from observed asset prices because assets prices also reflect the perceived likelihood of future government interventions, so the government cannot use the prices to learn about what asset values would be without intervention. Credit default swap prices of US banks during the financial crisis of 2007 to 2009 provide clear evidence of this issue.

14In practice, there might be a lower bound on $\varepsilon$ because the government might not want to own the banks. An approximate optimal program could then be implemented at this lower bound $\varepsilon$. Similarly, the rate $h$ is chosen to rule out inefficient participation (the NIP constraint). In theory, any $h > 0$ would work, but in practice, parameter uncertainty could prevent $h$ from being too close to zero.
Four properties of this optimal program are worth mentioning. First, we use preferred stock because it is junior to new lenders at time 1 and senior to common equity, but the program could also be implemented with a subordinated loan. Second, it is important that the government also takes a position that is junior to equity holders. The warrants give the upside to the government, which limits opportunistic participation. Third, the use of warrants limits risk shifting incentives since the government owns the upside, not the old equity holders (see, for instance, Green (1984)). Fourth, the use of warrants may allow the government to credibly commit to protecting new equity holders. This may be important for reasons outside the model if investors worry about outright nationalization of the banks.

We also note that our results describe the optimal intervention for a given investment set. The optimal investment set is the solution to the government objection function (14) and depends on the distribution of asset values and investment opportunities $F(\varepsilon, v)$ and the deadweight losses of taxation $\chi$. We note that implementing the optimal investment set may require a menu of programs.

5 Extensions

In this section we present two extensions to our baseline model. We consider the consequences of heterogeneous assets within banks and of deposit insurance.\textsuperscript{15}

5.1 Heterogeneous assets within banks

We consider an extension of our model to allow for asset heterogeneity within banks. Suppose that the face value of assets at time 0 is $A + A'$. All these assets are ex-ante identical. At time 1, the bank learns which assets are $A'$ and which assets are $A$. The $A$ assets are just like before, with probability $p(\bar{a}, \varepsilon)$ of $A$ and $1 - p(\bar{a}, \varepsilon)$ of 0. The $A'$ assets are worth zero with certainty. The ex-ante problems are unchanged, so all programs are still equivalent at time 0.

The equity and debt guarantee programs are unchanged at time 1. So equity still dominates debt guarantee. But the asset purchase program at time 1 is changed. For any price $q > 0$ the banks will always want to sell their $A'$ assets. This will be true in particular

\textsuperscript{15}Other extensions to continuous asset distributions and the sale of safe assets are available upon request. They do not generate new insights that justify their inclusion in this section.
of the banks without profitable lending opportunities.

**Proposition 6** With heterogeneous assets inside banks, there is a strict ranking of programs: equity injection is best, debt guarantee is intermediate, asset purchase program is worse.

The main insight from this extension is that adverse selection across banks is different from adverse selection across assets within banks. Adverse selection within banks increases the cost of the asset purchase program but does not affect the other programs.

### 5.2 Deposit insurance

Suppose long term debt consists of two types of debt: deposits $\Delta$ and unsecured long term debt $B$ such that

$$ D = \Delta + B. $$

Suppose that the government provides insurance for deposit holders and that deposit holders have priority over unsecured debt holders. Then the payoffs are are:

$$ y^\Delta = \min (y, \Delta); y^B = \min (y - y^\Delta, B) $$

Deposits are safe if $\Delta < A + c_0$, and risky if $\Delta \geq A + c_0$.

**Proposition 7** With safe deposits, the cost and benefits of both time 0 and time 1 programs remain unchanged. With risky deposits, the costs of time 0 and time 1 programs decrease. The equivalence results and ranking of both time 0 and time 1 programs remain unchanged.

**Proof.** See Appendix. ■

If deposits are safe, banks always have sufficient income to repay deposit holders. Hence, the expected cost of deposit insurance is zero independent of whether there is a government intervention. As a result, the costs and benefits of all programs remain unchanged.

With risky deposits, the government has to pay out deposit insurance in the low-payoff state. Hence, every cash injection lowers the expected cost of deposit insurance in the low-payoff state one-for-one. As a result, the government recoups the cash injection both in the
high- and low-payoff state. Put differently, a cash injection represents a wealth transfer to depositors and, because of deposit insurance, a wealth transfer to the government. Hence, the equivalence results and the ranking of interventions remain unchanged.

6 Discussion of financial crisis of 2007/09

The financial crisis of 2007-2009 has underlined the importance of debt overhang. There is agreement among many observers that debt overhang is an important reason for the decline in lending and investment during the crisis (see Allen, Bhattacharya, Rajan, and Schoar (2008) and Fama (2009), among others).

For example, Ivashina and Scharfstein (2008) show that new lending was 68% lower in the three-month period around the Lehman bankruptcy relative to the three-month period before the Lehman bankruptcy. Using cross-sectional variation in bank access to deposit financing, the authors show that the reduction in lending reflects a reduction in credit supply by banks rather than a reduction in credit demand by borrowers.

The crisis has also shown the difficulty of finding effective solutions to the debt overhang problem. Several experts have expressed concerns that existing bankruptcy procedures for financial institutions are insufficient for reorganizing the capital structure. As an alternative, Zingales (2008) argues for a law change that allows for forced debt-for-equity swaps. Coates and Scharfstein (2009) suggest to restructure bank holding companies instead of bank subsidiaries. Ayotte and Skeel (2009) argue that Chapter 11 proceedings are adequate if managed properly by the government. Assuming that restructuring can be carried effectively, these approaches reduce debt overhang at low cost to the government. However, Swagel (2009) argues that the government lacks the legal authority to force restructuring and that changing bankruptcy procedures is politically infeasible once banks are in financial distress.

Moreover, concerns for systemic risk and contagion make it difficult to restructure financial balance sheets in the midst of a financial crisis. Aside from the costs of its own failure, the bankruptcy of a large financial institution may trigger further bankruptcies because of counterparty risks and runs by creditors. For example, Heider, Hoerova, and Holthausen (2008) emphasize the role of counterparty risk in the interbank market.

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The government may therefore decide to avoid restructuring because there is a positive probability of a breakdown of the entire financial system. Even if the government decides to let some institutions restructure, the government also has to address debt overhang among the financial institutions that do not restructure. In fact, even proponents of restructuring suggest to rank banks based on their financial health and only restructure banks below a cutoff. Hence, independent of whether the government restructures some banks, the optimal form of government intervention outside restructuring remains an important question.

Surprisingly however, while there is at least some agreement regarding the diagnostic (debt overhang), there is considerable disagreement about the optimal form of government intervention outside restructuring. The original bailout plan proposed by former Treasury Secretary Paulson favors asset purchases over other forms of interventions. Stiglitz (2008) argues that equity injections are preferable to asset purchases because the government can participate in the upside if financial institutions recover. Soros (2009a) also favors equity injections over asset purchases because otherwise banks sell their least valuable assets to the government. Diamond, Kaplan, Kashyap, Rajan, and Thaler (2008) argue that the optimal government policy should be a combination of both asset purchases and equity injections because asset purchases establish prices in illiquid markets and equity injections encourage new lending. Bernanke (2009) suggests that in addition to equity injections and debt guarantees the government should purchase hard-to-value assets to alleviate uncertainty about bank solvency. Geithner (2009) argues that asset purchases are necessary because they support price discovery of risky assets.

Other observers have pointed out common elements among the different interventions without necessarily endorsing a specific one. Ausubel and Cramton (2009) argue that both asset purchases and equity injections require to put a price on hard-to-value assets. Bebchuk (2008) argues that both asset purchases and equity injections have to be conducted at market values to avoid overpaying for bad assets. Soros (2009b) argues that bank recapitalization has to be compulsory rather than voluntary. Kashyap and Hoshi (2008) compare the financial crisis of 2007-2009 with the Japanese banking crisis and argue that in Japan both asset purchases and capital injections failed because the programs were too small. Scharfstein and Stein (2008) argue that government interventions should restrict banks from paying dividends because, if there is debt overhang, equity holders favor immediate
payouts over new investment. Acharya and Backus (2009) suggest that public lender of last resort interventions would be less costly if they borrowed some of the standard tools used in private contracts for lines of credit.

We believe our results in this paper make three contributions to this debate. First, we believe an analytical approach to this question is helpful because it allows the government to implement a principled approach in which financial institutions are treated equally and government actions are predictable. This approach is preferable to a trial-and-error approach in which the government adjust interventions depending on current market conditions and tailors interventions to requests of individual financial institutions. In fact, such a trial-and-error approach may create more uncertainty for private investors, which makes them even less willing to invest. Uncertainty also generates an option to wait for future interventions, which further undermines private recapitalizations. Moreover, tailor-made interventions are more likely to be influenced and distorted by powerful incumbents (see Hart and Zingales (2008), Johnson (2009)).

Second, we distinguish the economic forces that matter from the ones that do not by providing a benchmark in which the form of government interventions is irrelevant. Under symmetric information, all interventions implement the same level of lending at the same expected costs. In contrast, under asymmetric information buying equity dominates other forms of intervention because buying equity reduces the extent of adverse selection across banks. Our analysis also shows how the government can use warrants to minimize the expected cost to taxpayers, an important element which has not been emphasized in the public debate on the financial crisis. Interestingly, Swagel (2009) notes that the terms of the Capital Purchase Program, the first round of government intervention, consisted of providing a loan in exchange for preferred stock and warrants. This structure is qualitatively consistent with the optimal intervention.

Third, our analysis clarifies why government interventions are costly. Under symmetric information, equity holders are held to their participation constraint but debt holders receive an implicit transfer. Hence, the same economic force that generates debt overhang in the first place, also generate the cost to the government. Under asymmetric information, participating banks receive informational rents because otherwise they would choose not to participate. Hence, under asymmetric information government interventions are costly
because the government has to recapitalize at above market rates.

Our analysis focuses on one specific market failure, debt overhang, and its negative consequences on lending and investment. We have emphasized the importance of asymmetric information between the government and the private sector, but we have maintained the assumption of symmetric information within the private sector. Philippon and Skreta (2009) solve for the optimal form of intervention when the market failure is adverse selection among private agents. They find that debt guarantees are optimal and that the government should always aim for pooling interventions where all banks participate.

Finally, we note that our analysis does not address why the banking system entered financial distress and whether government bailouts affect future bank actions. In our model, we take debt overhang as given and rely on other research that links the financial crisis to securitization (Mian and Sufi (2008), Keys, Mukherjee, Seru, and Vig (2010)) and the tendency of banks to become highly levered (Adrian and Shin (2008), Acharya, Schnabl, and Suarez (2009), Kacperczyk and Schnabl (2010)). Regarding the impact of government interventions on future bank actions, we recognize that bailouts can create expectations of future bailouts which may cause moral hazard. However, if the government decides to intervene, then it is optimal for the government to choose the intervention with the lowest costs. Also, the optimal intervention minimizes rents to equity and debt holders, so the optimal intervention also minimizes moral hazard conditional on the decision to intervene.

7 Conclusion

In this paper we study the efficiency and welfare implications of different government interventions in a standard model with debt overhang. We consider asset purchases, equity injections, and debt guarantees. We find that under symmetric information, all interventions are equivalent. Under asymmetric information, equity injections dominate both asset purchases and debt guarantees. Under both symmetric and asymmetric information, the government can improve economic efficiency by requiring a minimum number of firms to participate. We also solve for the optimal mechanism, and find that it can be implemented with preferred stock and warrants.
References


Proof of Proposition 2

Cash injection

The government offers cash \( m \) against fraction \( \alpha \) of equity capital. The government recognizes that the equilibrium is \( \bar{a}(m) \) which yields the investment domain \( \bar{I}(m) \). At time 0, equity holders participate in the voluntary intervention if

\[
(1 - \alpha) E_0[y^o|\bar{a}, m] \geq E_0[y^o|\bar{a}, 0].
\]  

(24)

The cost of the program to the government is

\[
\Psi^e_0(m, \alpha) = m - \alpha E_0[y^o|\bar{a}, m].
\]

Because the investment domain does not depend on \( \alpha \), the government chooses equity share \( \alpha \) such that the participation constraint (24) binds. Using the participation constraint (24) to eliminate \( \alpha \) from the cost function yields

\[
\Psi^e_{0,\text{free}}(m, \alpha) = m - (E_0[y^o|\bar{a}, m] - E_0[y^o|\bar{a}, 0]).
\]

Using expected shareholder value at time 0

\[
E_0[y^o|\bar{a}, m] - E_0[y^o|\bar{a}, 0] = \bar{p}(\bar{a}) m + m \int_{\bar{I}(m)} \int_{\bar{I}(m) \setminus \bar{I}(\bar{a}, 0)} (1 - p(\bar{a}, \varepsilon)) dF(\varepsilon, v) + \int_{\bar{I}(m) \setminus \bar{I}(\bar{a}, 0)} (p(\bar{a}, \varepsilon) v - x) dF(\varepsilon, v).
\]

Therefore the cost of the government is

\[
\Psi^e_{0,\text{free}}(m, \alpha) = m - \alpha E_0[y^o|\bar{a}, m]
\]

\[
= (1 - \bar{p}(\bar{a})) m - m \int_{\bar{I}(m)} \int_{\bar{I}(m) \setminus \bar{I}(\bar{a}, 0)} (1 - p(\bar{a}, \varepsilon)) dF(\varepsilon, v) - \int_{\bar{I}(m) \setminus \bar{I}(\bar{a}, 0)} (p(\bar{a}, \varepsilon) v - x) dF(\varepsilon, v)
\]

\[
= \Psi^e_{0,\text{free}}(m)
\]

Debt guarantee

The government recognizes that the equilibrium conditional on intervention is given by the function \( \bar{a}((1 - \phi)S) \). Using equation (15), we see that conditional on participation, the equity value at time 0 is \( E_0[y^o|\bar{a}, (1 - \phi)S] - \bar{p}(\bar{a})S \). If a bank opts out, equity value becomes \( E_0[y^o|\bar{a}, 0] \). Since participation only depends on \( m = (1 - \phi)S \), the government chooses the program such that the participation constraint binds:

\[
\bar{p}(\bar{a})S = \bar{p}(\bar{a}) m + (1 - \phi)S \int_{\bar{I}(m)} (1 - p(\bar{a}, \varepsilon)) dF(\varepsilon, v) + \int_{\bar{I}(m) \setminus \bar{I}(\bar{a}, 0)} (p(\bar{a}, \varepsilon) v - x) dF(\varepsilon, v)
\]

The cost to the government is

\[
\Psi^e_{0,\text{free}} = (1 - \bar{p}(\bar{a})) S - \phi S
\]

Plugging the participation constraint into the cost function yields the expected cost \( \Psi^e_{0,\text{free}}(m) \) defined in equation (16). The program is equivalent to an asset purchase program when \( Zq = (1 - \phi)S \).
Proof of Proposition 4

We omit $\bar{a}$ to shorten the notations but all the calculations are conditional on the equilibrium value of $\bar{a}$. We must show equivalence along four dimensions: (i) the NIP constraint, (ii) the upper schedule, (iii) the lower schedule, and (iv) the cost function. Upon participation and investment, equity value is

$$E_1[y^e|i = 1; S, \phi] = p(\varepsilon)(A - D) + p(\varepsilon)v - x + (1 - \phi - p(\varepsilon))S$$

Participation without investment yields

$$E_1[y^e|i = 0; S, \phi] = p(\varepsilon)(A - D - \phi S).$$

Now consider the three constraints:

- NIP: $E_1[y^e|i = 0; S, \phi] < E_1[y^e|i = 0; 0, 0]$ or $\phi > 0$.

- Upper schedule: $E_1[y^e|i = 1; S, \phi] > E_1[y^e|i = 1; 0, 0]$ or

  $$U_1(\varepsilon, v; S, \phi) = (1 - \phi - p(\varepsilon))S.$$

- Lower schedule: $E_1[y^e|i = 1; S, \phi] > E_1[y^e|i = 0; 0, 0]$ or

  $$L_1(\varepsilon, v; S, \phi) = p(\varepsilon)v - x + (1 - \phi - p(\varepsilon))S$$

Using the notations of the asset purchase program, the participation set is $\Omega^*_1(S, 1 - \phi)$, the investment domain is $I^*_1(S, 1 - \phi)$ and the expected cost of the program is

$$\Psi_1(S, 1 - \phi) = \phi S - \int_{\Omega^*_1(S, 1 - \phi)} \int (1 - p(\varepsilon)) dF(\varepsilon, v)$$

Now if we set $S = Z$ and $q = 1 - \phi$, we see that the NIP constraint, the upper and lower schedules, and the cost functions are the same as for the asset purchase program. The two programs are therefore equivalent.

Proof of Proposition 5

We omit $\bar{a}$ to shorten the notations but all the calculations are conditional on the equilibrium value of $\bar{a}$. We first analyze the equity injection program at time 1. Upon participation and investment, equity value (including the share going to the government) is

$$E_1[y^e|i = 1; m] = p(\varepsilon)(A - D) + p(\varepsilon)v - x + m$$

Participation without investment yields

$$E_1[y^e|i = 0; m] = p(\varepsilon)(A - D + m)$$

Now consider the three constraints
• NIP: \((1 - \alpha) E_1[y^e|i = 0; m] < E_1[y^e|i = 0,0]\) or:
\((1 - \alpha) m < \alpha (A - D)\).

• Upper schedule: \((1 - \alpha) E_1[y^e|i = 1; m] - E_1[y^e|i = 1;0]\) or:
\(U^e_1 = (1 - \alpha) m - \alpha (p(\varepsilon) (A - D) + p(\varepsilon) v - x)\).

• Lower schedule: \((1 - \alpha) E_1[y^e|i = 1; m] - E_1[y^e|i = 0;0]\) or:
\(L^e_1 = (1 - \alpha) (p(\varepsilon) v - x + m) - \alpha p(\varepsilon) (A - D)\).

If we define the function \(X(\varepsilon; m, \alpha) \equiv (1 - \alpha) m - \alpha p(\varepsilon) (A - D)\) as in equation (23), we can rewrite the program as:
\[L^e_1 = (1 - \alpha) (p(\varepsilon) v - x) + X(\varepsilon; m, \alpha)\]
\[U^e_1 = \alpha (p(\varepsilon) v - x) - X(\varepsilon; m, \alpha)\]

The participation set is
\[\Omega^e_1(m, \alpha) = \{(\varepsilon, v) \mid L^e_1 > 0 \land U^e_1 > 0\} \ .\]

The cost function is therefore
\[\Psi^e_1(m, \alpha) = \int_{\Omega^e_1(m, \alpha)} (m - \alpha E_1[y^e|i = 1; m]) dF(\varepsilon, v) .\]

We can rewrite the cost function such that
\[\Psi^e_1(m, \alpha) = \int_{\Omega^e_1(m, \alpha)} X(\bar{a}, \varepsilon; m, \alpha) dF(\varepsilon, v) - \alpha \int_{\Omega^e_1(m, \alpha)} (p(\varepsilon) v - x) dF(\varepsilon, v) .\]

The following table provides a comparison of the government interventions:

<table>
<thead>
<tr>
<th></th>
<th>Asset purchase</th>
<th>Equity injection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participation</td>
<td>(\Omega^e_1(\bar{a}, Z, q))</td>
<td>(\Omega^e_1(\bar{a}, m, \alpha))</td>
</tr>
<tr>
<td>Investment</td>
<td>(I(\bar{a}) \cup \Omega^e_1(\bar{a}, Z, q))</td>
<td>(I(\bar{a}) \cup \Omega^e_1(\bar{a}, m, \alpha))</td>
</tr>
<tr>
<td>NIP constraint</td>
<td>(q &lt; 1)</td>
<td>((1 - \alpha) m &lt; \alpha (A - D))</td>
</tr>
<tr>
<td>Cost function</td>
<td>(\Psi^e_1(\bar{a}, Z, q))</td>
<td>(\Psi^e_1(\bar{a}, m, \alpha))</td>
</tr>
</tbody>
</table>

Now let us prove that the equity injection program dominates the other two programs. Take an asset purchase program \((Z, q)\). We are going to construct an equity program that
The preferred stock is paid first. Then shareholders receive

\[ L^q_\varepsilon (\varepsilon, v; m, \alpha) = L^q_1 (\varepsilon, v; q, Z) \] for all \( \varepsilon, v. \)

It is easy to see that this is indeed possible if we identify term by term: \( \frac{\alpha}{1 - \alpha} = \frac{Z}{A - D} \) and \( m = qZ \). In this case we also have \( I^q_1 (\bar{a}, S, \phi) = I^q_1 (\bar{a}, m, \alpha) \). The NIP constraint are also equivalent since \((1 - \alpha) m < \alpha (A - D) \iff q < 1.\)

Now consider the upper bound. Consider the lowest point on the upper schedule of the asset purchase program, i.e., the intersection of \( U^{e}_1 = 0 \) with \( p(\varepsilon) v - x = 0 \). At that point \((\bar{\varepsilon}, \bar{v}), \) we have \( p(\bar{\varepsilon}) = q \) and \( \bar{v} = x/q \). Using the fact that lower bounds are equal to zero, we can write \( X (\varepsilon; m, \alpha) = (1 - \alpha) (1 - p(\varepsilon)) qZ \) for all \( \varepsilon, v \). This implies that \( X (\bar{\varepsilon}; m, \alpha) = 0, \) and therefore \( U^{e}_1 (\bar{\varepsilon}, \bar{v}; m, \alpha) = \alpha (p(\bar{\varepsilon}) \bar{v} - x) - X (\bar{\varepsilon}, \bar{v}; m, \alpha) = 0. \) Therefore the upper schedule \( U^{e}_1 (p, v; m, \alpha) = 0 \) also passes by this point. But the schedule \( U^{e}_1 (p, v; m, \alpha) = 0 \) is downward slopping in \((\varepsilon, v), \) so the domain of inefficient participation is smaller (see Figure 7) than in the asset purchase case. Formally, we have just shown that:

\[ \Omega^{e}_1 (m, \alpha) < \Omega^q_1 (S, \phi). \]

As an aside, it is also easy to see that the schedule \( U^{e}_1 (p, v; m, \alpha) = 0 \) is above the schedule \( pv - x = 0 \) so it does not get rid completely of opportunistic participation, but it helps.

The final step is to compare the cost functions \( \Psi^{e}_1 (q, Z) \) and \( \Psi^{e}_1 (m, \alpha). \) By definition of the participation domain, we know that lower bound \( L^{e}_1 (\bar{a}, \varepsilon, v; m, \alpha) > 0. \) Therefore:

\[ -\int \int_{\Omega^{e}_1 (\bar{a}, m, \alpha)} (p(\varepsilon) v - x) dF(p, v) < \frac{X(\varepsilon; m, \alpha)}{1 - \alpha} \text{ for all } (\varepsilon, v) \in \Omega^{e}_1 (m, \alpha) \]

Therefore

\[ \Psi^{e}_1 (m, \alpha) < \frac{1}{1 - \alpha} \int \int_{\Omega^{e}_1 (\bar{a}, m, \alpha)} X(\varepsilon; m, \alpha) dF(\varepsilon, v) = Z \int \int_{\Omega^{e}_1 (\bar{a}, m, \alpha)} (q - p(\varepsilon)) dF(\varepsilon, v) \]

Since \( q - p(\varepsilon) > 0 \) for all \( \varepsilon, v \in \Omega^{e}_1 (m, \alpha), \) and since \( \Omega^{e}_1 (m, \alpha) \subset \Omega^q_1 (q, Z), \) we have

\[ \Psi^{e}_1 (m, \alpha) < \Psi^q_1 (q, Z). \]

Finally, note that the the comparison is conditional on equilibrium \( \bar{a}. \) However, the equity injection requires lower taxes and therefore lead to higher equilibrium level \( \bar{a}. \)

**Proof of Theorem 2**

In the good state, the residual payoffs conditional on investment are

\[ A - D + \frac{p(\varepsilon) v - x + m}{p} \]

The preferred stock is paid first. Then shareholders receive

\[ y^e_i = \max \left( A - D + \frac{p(\varepsilon) v - x + m}{p} - (1 + h) m, 0 \right) \text{ if } i = 1 \text{ and } a = A \]

\[ y^e_i = \max \left( A - D - (1 + h) m, 0 \right) \text{ if } i = 0 \text{ and } a = A. \]
As soon as \( y^e > A - D \), the warrants are in the money and the number of shares jumps to \( 1 + \frac{1 - y}{\eta} = \frac{1}{\eta} \). So the old shareholders get only a fraction \( \eta \) of the value beyond \( A - D \). Their payoff function is therefore:

\[
f(y^e) = \min (y^e, A - D) + \eta \max (y^e - A - D, 0).
\]

So old shareholders are full residual claimants up to the face value of old assets \( A - D \) and residual claimants beyond. Now let us think about their decisions at time 1. As usual only the payoffs in the non default state matter. If they do not invest they get \( A - D \). They receive more by investing if and only if

\[
 v(x_{\text{m}}) p(1 + h) > 0
\]

The lower participation constraint is therefore

\[
(p(\varepsilon) v - x) + m (1 - (1 + h) p(\varepsilon)) > 0.
\]

It converges to \( L(\varepsilon, v) + (1 - p(\varepsilon)) m \) if \( h \to 0 \). We can compare this to the equity injection schedule \( L^e_1(\varepsilon, v; m, \alpha) \), we can identify the same cash injection \( m \), and the ratio of new shareholders to old shareholders

\[
\frac{\alpha}{1 - \alpha} = \frac{m (1 + h)}{A - D}.
\]

If we compare to debt guarantee \( L^d_1(\varepsilon, v; Z, q) = p(\varepsilon) v - x + Z(q - p(\varepsilon)) \). Then

\[
m = qZ \text{ and } q = \frac{1}{1 - h}.
\]

Next consider the upper schedule. Investing alone gets \( A - D + (p(\varepsilon) v - x) / p \) so they opt in if and only if \( p(\varepsilon) v - x > \eta (p(\varepsilon) v - x + (1 - (1 + h) p) m) \) and therefore

\[
U = p(\varepsilon) v - x - m\eta \frac{1 - (1 + h) p}{1 - \eta}.
\]

The upper bound converges to \( p(\varepsilon) v - x \) when \( \eta \to 0 \). The NIP constraint is simply

\[
h > 0.
\]

Finally, the cost of the program is small because the government gets all the upside value of the new projects. The expected payments to the old shareholders converge to \( p(A - D) \). So the government receives expected value \( p(\varepsilon) v - x + m \) by paying \( m \) at time 1. The total cost is therefore:

\[
-\int_{I(m) \setminus I(0)} (p(\varepsilon) v - x) dF(\varepsilon, v)
\]

The cost is positive because \( p(\varepsilon) v < x \) for all \( (\varepsilon, v) \in I(m) \setminus I(0) \).

**Proof of Proposition 7**

If deposits are safe, then the optimization problem from the equity holders perspective remains unchanged because the investment and participation decision only depend on total debt \( D \). Now consider the expected cost of deposit insurance. Note that the time 0 expected value of deposits is \( \Delta \) because \( \Delta \leq A \). Hence, the cost of government intervention is unchanged and therefore the cost and benefits of both time 0 and time 1 programs are unchanged. If the deposits are risky, we distinguish two cases depending on the recovery rate on deposits in the low payoff state. We omit \( \tilde{a} \) to shorten the notations but all the calculations are conditional on the equilibrium value of \( \tilde{a} \).
Time 0 Programs

Full transfer: $\Delta + v < \Delta$

The expected values of deposits at time 1 and time 0 are

$$E_1 [y^\Delta (m)] = p(\varepsilon) \Delta + (1 - p(\varepsilon))(\Delta + m) \text{ if } (\varepsilon, v) \in T \setminus I(m)$$

$$E_0 [y^\Delta (m)] = p(\varepsilon) \Delta + (1 - p(\varepsilon))(\Delta + v) \text{ if } (\varepsilon, v) \in I(m)$$

$$E_0 [y^\Delta (m)] = \bar{p}\Delta + (1 - \bar{p})(\Delta + m) + \int \int_{I(m)} (1 - p)(v - m) \, dF(\varepsilon, v).$$

The expected cost of deposit insurance at time 0 is

$$\Psi^F_0 (m) = \Delta - E_0 [y^\Delta (m)]$$

$$= (1 - \bar{p})(\Delta - \Delta - m) - \int \int_{I(m)} (1 - p(\varepsilon))(v - m) \, dF(\varepsilon, v)$$

The change in the expected cost of deposit insurance is

$$\Lambda^F_0 (m) = \Psi^F_0 (m) - \Psi^F_0 (0)$$

$$= -(1 - \bar{p}) m + m \int \int_{I(m)} (1 - p) \, dF(\varepsilon, v) - \int \int_{I(m) \setminus I^o} (1 - p(\varepsilon)) \, vdF(\varepsilon, v).$$

The net cost of government intervention is

$$\bar{\Psi}_0 (m) + \Lambda^F_0 (m) = - \int \int_{I(m) \setminus I^o} vdF(\varepsilon, v).$$

Note that this term is negative because the benefits of incremental investments accrue to the government.

Partial Transfer: $\Delta + c_0 < \Delta < \Delta + v$

The expected values of deposits at time 1 and time 0 are

$$E_1 [y^\Delta (m) | \varepsilon, v] = p(\varepsilon) \Delta + (1 - p(\varepsilon)) \max(\Delta, \Delta + m) \text{ if } (p, v) \in T \setminus I_0(m)$$

$$E_0 [y^\Delta (m)] = \Delta - \int \int_{T \setminus I_0(m)} (1 - p(\varepsilon))(\Delta - \max(\Delta, \Delta + m)) \, dF(\varepsilon, v)$$

The expected cost of deposit insurance is

$$\Psi^F_0 (m) = \int \int_{T \setminus I_0(m)} (1 - p(\varepsilon))(\Delta - \max(\Delta, \Delta + m)) \, dF(\varepsilon, v).$$
The change in the expected cost of deposit insurance

\[ \Lambda^F_0 (m) = \int_{T \setminus I_0(m)} \int (1 - p(\varepsilon)) (\Delta - \max(\Delta, A + m)) p(\varepsilon) - \int_{T \setminus I_0} \int (1 - p(\varepsilon)) (\Delta - A) dF(\varepsilon, v). \]

Note that when \( \Delta \to A \), then \( \Lambda^F_0 (m) \to 0 \). This means the expected change in the cost of deposit insurance goes to zero as deposits become safe. Also note that when \( \Delta \to (A + v) \), then

\[ \Lambda^F_0 (m) \to -(1 - \bar{p}) m + m \int_{I(m)} \int (1 - p(\varepsilon)) dF(\varepsilon, v) - \int_{I(m) \setminus I_0} \int (1 - p(\varepsilon)) vdF(\varepsilon, v) \]

which is the change in expected cost of deposit insurance in the full transfer case. The government cost is \( \Lambda^F_0 (m) + \Psi^F_0 (m) \). The results apply to all programs because all programs have the same cost function at time 0.

**Time 1 programs**

**Full Transfer: \( A + v < \Delta \)**

The expected values of deposits at time 1 and time 0 are

\[ E_1[y^\Delta(Z,q)] = p(\varepsilon) \Delta + (1 - p(\varepsilon)) A \text{ if } (\varepsilon, v) \in T \setminus (I_0 \cup \Omega^q(Z, 1 - q)) \]

\[ = p(\varepsilon) \Delta + (1 - p(\varepsilon)) A + v \text{ if } (\varepsilon, v) \in I_0 \cup \Omega^q(Z, 1 - q) \]

\[ E_0[y^\Delta(Z,q)] = \bar{p} \Delta + (1 - \bar{p}) A + \int_{I_0 \cup \Omega^q(Z, 1 - \bar{p})} \int (1 - p(\varepsilon)) vdF(\varepsilon, v) \]

The expected cost of deposit insurance is

\[ \Psi^F_0(Z,q) = (1 - \bar{p}) (\Delta - A) - \int_{I_0 \cup \Omega^q(Z, 1 - q)} \int (1 - p(\varepsilon)) vdF(\varepsilon, v) \]

The change in the cost of deposit insurance is

\[ \Psi^F_0(Z,q) = - \int_{\Omega^q(Z, 1 - q) / I_0} \int (1 - p(\varepsilon)) vdF(\varepsilon, v) \]

Expected government cost is

\[ \Psi^a_1(Z,p^\varepsilon) = Z \int_{\Omega^q(Z, 1 - p^\varepsilon)} \int (q - p(\varepsilon)) dF(p,v) \]

\[ = \Lambda(Z, 1 - q) - \int_{\Omega^q(Z, 1 - q) / I_0} \int (1 - p(\varepsilon)) vdF(\varepsilon, v). \]
Partial Transfer: $A < \Delta < A + v$

The expected values of deposits at time 1 and time 0 are

$$E_1 [y^\Delta (Z, q)] = \begin{cases} p\Delta + (1 - p(\varepsilon)) \Delta & \text{if } (\varepsilon, v) \in T \setminus (I^o \cup \Omega^q_1 (Z, 1 - q)) \\ \Delta & \text{if } (\varepsilon, v) \in I^o \cup \Omega^q_1 (Z, 1 - q) \end{cases}$$

$$E_0 [y^\Delta (Z, q)] = \Delta - \int_{T \setminus (I^o \cup \Omega^q_1 (Z, 1 - q))} (1 - p(\varepsilon)) (\Delta - A) \, dF(\varepsilon, v)$$

The expected cost of government insurance is

$$\Psi_0^F (Z, q) = \int_{T \setminus (I^o \cup \Omega^q_1 (Z, 1 - q))} (1 - p(\varepsilon)) (\Delta - A) \, dF(\varepsilon, v).$$

The change in expected cost of deposit insurance is

$$\Lambda_0^F (Z, q) = - \int_{\Omega^q_1 (Z, 1 - p^z) / I^o} (1 - p(\varepsilon)) (\Delta - A) \, dF(\varepsilon, v).$$

Note that when $\Delta \to A$, then $\Lambda_0^F (Z, q) \to 0$. Also note that when $\Delta \to (A + v)$, then

$$\Lambda_0^F (Z, q) \to - \int_{\Omega^q_1 (Z, 1 - p^z) / I^o} (1 - p(\varepsilon)) (v - c_0) \, dF(\varepsilon, v).$$

Total government cost is

$$\Psi_1^q (Z, q) = Z \int_{\Omega^q_1 (Z, 1 - p^z)} (q - p(\varepsilon)) \, dF(p, v)$$

$$= \Lambda (Z, 1 - q) - \int_{\Omega^q_1 (Z, 1 - p^z) / I^o} (1 - p(\varepsilon)) (\Delta - A) \, dF(\varepsilon, v).$$

The results also apply to debt guarantees at time 1 because asset purchases and debt guarantees have the same cost function at time 1.

**Cash against equity at time 1**

Note that we can compute the expected cost of time 1 cash against equity similarly to the time 1 asset purchase program. The only difference is the participation region for cash against equity $\Omega^c (m, \alpha)$ and the participation region for asset purchase $\Omega^q_1 (Z, 1 - q)$. It turns out that the change in the expected cost of deposit insurance $\Lambda_0^F (m)$ is equivalent under both programs because both in the full and partial transfer case the difference in the participation region cancels out when computing the difference in expected cost of deposit insurance. It follows that the relative ranking of programs is unchanged.
Fig 1: Information & Technology

$t = 0$

$\text{Existing Assets}$

$p$

$A + A$

$t = 1$

$1 - p$

$A + 0$

$t = 2$

$-x$

$\nu$
## Fig 2: Payoffs

### $t = 1$

<table>
<thead>
<tr>
<th>Learn $p$ and $v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_2 = c_1$</td>
</tr>
<tr>
<td>Keep cash</td>
</tr>
<tr>
<td>Invest</td>
</tr>
<tr>
<td>$c_2 = c_1 + l - x$</td>
</tr>
</tbody>
</table>

### $t = 2$

<table>
<thead>
<tr>
<th>Senior Debt</th>
<th>New Debt</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>$A+c_1$</td>
<td>$A+D+D+c_1$</td>
</tr>
<tr>
<td>$1-p$</td>
<td>$A+c_1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$p$</td>
<td>$D$</td>
<td>$A+D+D+c_2+v-rl$</td>
</tr>
<tr>
<td>$1-p$</td>
<td>$A+c_2+v$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

*Keep cash and Invest branches take into account the decision whether to keep cash or invest in the next period.*
Fig 3: First Best

$\nu \leq \nu^*$

$x \leq v = x$
Fig 4: Debt Overhang

\[ V \]

\[ L^0 = 0 \]

\[ I^0 \]
Fig 5: Cash at time 0

\[ L^0 + (1-p)m = 0 \quad L^0 = 0 \]

\[ I(m) \]
Fig 6: Asset Purchase at time 1

Efficient participation

Opportunistic participation

Invest alone
Figure 7: Equity injection at time 1

\[ L^e(m,\alpha) = 0 \]

\[ U^e(m,\alpha) = 0 \]

\[ L^o = 0 \]
Figure 8: Efficient Mechanism

\[ L(\Gamma) = 0 \]

\[ U(\Gamma) = 0 \]

\[ L^0 = 0 \]